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A KEY

TO

ELEMENTARY STATICS AND HYDROSTATICS

BY

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AND LATE LECTURER AT ST. PETER'S COLLEGE, CAMBRIDGE

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NOTE.

IN a few instances the solutions in the Key to the Statics do not agree with the questions not correctly stated in the last edition (1875).

ELEMENTARY STATICS.

KEY.

EXAMPLES—I. (p. 25).

1. Let C be the body. ACB a straight line.

Then if the three forces act all in the same direction along CB, a force of 12 lbs. will be required acting along CA to keep them at rest.

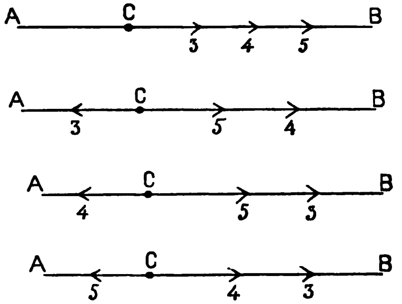

FIG. 1.

If the forces of 5 lbs. and 4 lbs. act along CB, and the force of 3 lbs. along CA, a force of 6 lbs. will be required acting along CA to keep equilibrium ; and so for the other cases.

2. As in the preceding example, if the forces all act along CB they will have a resultant acting along CB of 29 lbs.

If the forces of 11 lbs. and 13 lbs. act along CB, and the force of 5 lbs. along CA, they will have a resultant of 19 lbs. acting along CB ; and so for the other cases.

3. Let x be the measure of the magnitude of the resultant in lbs.

$$\text{Then } x^2=(90)^2+(120)^2$$
$$=8100+14400=22500\,;$$
$$\therefore\ x=150, \text{ and the resultant is one of 150 lbs.}$$

4. Here $x^2 = (36)^2 + (48)^2$
$= 1296 + 2304 = 3600$;
$\therefore x = 60$, and the resultant is one of 60 lbs.

5. As the squares of the numbers 6 and 8, that is, 36 and 64, are together equal to 100, which is the square of 10, it is plain from the first diagram on page 23 of the *Statics*, that if the measures of AC, AB be 8 and 6, and the measure of AD be 10, the square on AD will be equal to the sum of the squares on AC, CD, and, therefore, $\angle ACD$ will be a right angle (EUCLID, I. XLVIII) ; whence $\angle BAC$ is also a right angle (EUCLID, I. XXIX).

6. Since the squares of 3 and 4, that is, 9 and 16, are together equal to 25, which is the square of 5, the forces 3 and 4 will act at right angles to each other. Hence if C be the point, draw CA, CB containing 3 and 4 units of length respectively, complete the parallelogram $ACBD$. Produce DC to E, making $CE = DC = 5$ units. Then CA, CB, CE will represent the arrangement of the three forces.

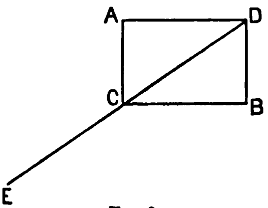

FIG. 2.

7. Let the measures of the forces be x and $\sqrt{3} . x$, which numbers are in the ratio of $1 : \sqrt{3}$.

Then $x^2 + (\sqrt{3} . x)^2 = (10)^2$;
or, $x^2 + 3x^2 = 100$; or, $4x^2 = 100$; or, $x = 5$;
$\therefore$ the forces are 5 lbs. and $5\sqrt{3}$ lbs.

8. Construct a diagram as in Example 2 on p. 23, but with $\angle BAC = 30°$.

Then $\angle DCE = 30°$, $\angle CDE = 60°$, and $CD = 2DE$.

Hence $CE^2 = CD^2 - DE^2$

$$= CD^2 - \frac{CD^2}{4} = \frac{3CD^2}{4};$$

$$\therefore CE = \frac{\sqrt{3} . CD}{2}.$$

Then if x be the measure of AD,

$$x^2=3^2+5^2+2\,.\,3\,.\,\frac{\sqrt{3}.5}{2}$$

$$=9+25+15\sqrt{3}=34+15\sqrt{3};$$

$$\therefore x=\sqrt{34+15\sqrt{3}}.$$

Or, by Trigonometry,

$$x^2=3^2+5^2+2\,.\,3\,.\,5\,.\cos 30^\circ=9+25+30\,.\,\frac{\sqrt{3}}{2}=34+15\sqrt{3}.$$

9. Construct a diagram as in Example 2 on p. 23.

$$\text{Then } x^2=10^2+7^2+2\,.\,10\,.\,\frac{7}{2}$$

$$=100+49+70=219;$$

$$\therefore x=\sqrt{219}.$$

Or, by Trigonometry,

$$x^2=10^2+7^2+2\,.\,10\,.\,7\,.\cos 60^\circ=100+49+140\,.\,\frac{1}{2}=219.$$

10. Let AB, AC represent the forces, $\angle BAC$ being an angle of 135°.

Complete the parallelogram $ABDC$, draw DE perpendicular to AC.

Then $\angle ACD=45^\circ$ (EUCLID, I. XXIX.), and $EC=DE$.

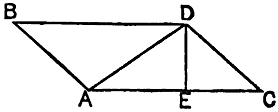

FIG. 3.

Also, $CD^2=DE^2+EC^2$

$=2EC^2$,

and $\therefore EC=\dfrac{CD}{\sqrt{2}}$.

Hence, if x be the measure of AD,

$$x^2=AC^2+CD^2-2AC\,.\,EC. \qquad \text{(EUCLID, II. XIII.)}$$

$$=11^2+9^2-2\,.\,11\,.\,\frac{9}{\sqrt{2}}=121+81-99\sqrt{2}.=202-99\sqrt{2};$$

$$\therefore x=\sqrt{202-99\sqrt{2}}.$$

Or, by Trigonometry,

$$x^2=11^2+9^2+2\,.\,11\,.\,9\,.\cos 135^\circ$$

$$=121+81+198\times\left(-\frac{1}{\sqrt{2}}\right)=202-99\sqrt{2}.$$

11. Construct a diagram as in Example 2 of p. 23, but having $\angle BAC=45°$, and $\therefore \angle DCE=45°$.

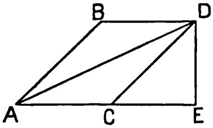

FIG. 4.

Then, as in Example 10, $CE=\frac{CD}{\sqrt{2}}$.

Hence, if x be the measure of AD

$$x^2=AC^2+CD^2+2AC.CE$$
$$=4^2+5^2+2.4.\frac{5}{\sqrt{2}}$$
$$=16+25+20\sqrt{2}=41+20\sqrt{2};$$
$$\therefore x=\sqrt{41+20\sqrt{2}}.$$

Or, by Trigonometry,

$$x^2=4^2+5^2+2.4.5.\cos 45°$$
$$=16+25+40.\frac{1}{\sqrt{2}}=41+20\sqrt{2}.$$

12. This Example is to be worked precisely in the same way as Example 2 on p. 23, except that instead of 8 lbs. we must put F to represent each of the given forces. Then the magnitude of the resultant will be $F\sqrt{3}$ in place of $8\sqrt{3}$ lbs.

Or, by Trigonometry,

$$x^2=F^2+F^2+2F^2.\cos 60°$$
$$=2F^2+2F^2.\frac{1}{2}=3F^2.$$

13. Let AB, AC represent the forces acting at A.

Produce BA to E, making $AE=AB$.

Complete the parallelogram $ABDC$, $AEFC$.

Then, if P and Q be the forces represented by AB and AC, and if $\angle BAC=a$, and $\therefore \angle EAC=180°-a$, and if R and R' be the resultants in each case,

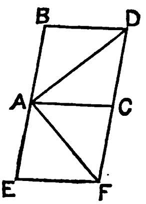

FIG. 5.

$$R^2=P^2+Q^2+2PQ.\cos a,$$
$$R'^2=P^2+Q^2-2PQ.\cos a;$$
$$\therefore R^2+R'^2=2(P^2+Q^2).$$

But since DAF is a right-angled triangle,

$$FD^2=AD^2+AF^2;$$
$$\therefore (2P)^2=R^2+R'^2$$
$$=2(P^2+Q^2);$$
$$\therefore 2P^2=2Q^2,$$

which cannot be unless $P=Q$.

14. Let D be the point where the vertical line of direction, in which the weight acts, meets a line drawn from A parallel to BF.

Then the component forces, which are F and the tension of the string BA, being represented by BC and BA, BD will represent the resultant force of 10 lbs.

Now, DBF is a right angle, and $\angle DCB = 60°$. (EUCLID, I. XXIX.)

$$\therefore BD = \sqrt{3} . BC;$$

$$\therefore F = \frac{10}{\sqrt{3}} \text{ lbs.} = \frac{10\sqrt{3}}{3} \text{ lbs.}$$

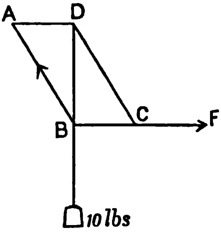

FIG. 6.

EXAMPLES—II. (p. 31).

1. As in Example 12 of I., we find the resultant of P and P to be $P . \sqrt{3}$; and the resultant of Q and Q is $\sqrt{Q^2 + Q^2}$, or $Q . \sqrt{2}$;

$\therefore$ since the resultants are equal,

$$P\sqrt{3} = Q\sqrt{2}, \text{ that is, } P : Q = \sqrt{2} : \sqrt{3}.$$

2. Let P and Q be the forces, and let R, their resultant, be equal to P. Draw a diagram as in Art. 38.

Then since $AB = AD$, $\therefore \angle ABD = \angle ADB$.

Now $\angle ABD = 60°$, because $\angle BAC = 120°$;

$\therefore \angle ABD$ and $\angle ADB$ are each $= 60°$;

$\therefore \angle BAD = 60°$ (EUCLID, I. XXXII.)

that is, the triangle ABD is equilateral, and $\therefore P = Q$.

3. (1) The proof depends on the fact that the diagonals of a parallelogram bisect each other. See Art. 34, 3.

Hence FE is half the diagonal of a parallelogram, of which AE, DE are the adjacent sides.

And EF is half the diagonal of a parallelogram, of which BF, CF are the adjacent sides.

Hence resultant of AE, $DE = 2FE$,

and resultant of BF, $CF = 2EF$;

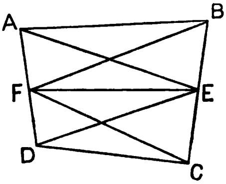

FIG. 7.

$\therefore$ since $2FE$ and $2EF$ represent equal and opposite forces, forces represented by AE, DE, BF, CF are in equilibrium.

(2) The length of a side of the square being S, the length of the diagonal is $\sqrt{S^2+S^2}$, or $S\sqrt{2}$.

Now the resultant of forces S and $S\sqrt{2}$ acting at an angle of 45° (for the diagonal bisects the angles of the square), is found, as we proved in Example 11 of I., thus—

$$x^2 = S^2 + 2S^2 + 2 \cdot S \cdot \frac{S\sqrt{2}}{\sqrt{2}}$$

$$= S^2 + 2S^2 + 2S^2 = 5S^2.$$

4. If AB, AC represent the two equal forces, and AD the third force, $\angle BAC = 60°$, because the sum of $\angle$s BAD, $CAD = 300°$.

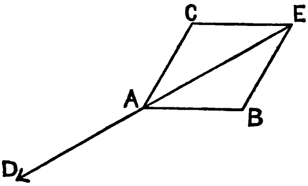

FIG. 8.

Then, as in Example 12 of I.,

$$AE = \sqrt{3} \cdot AB.$$

Hence if F be the magnitude of each of the equal forces, the magnitude of the third force is $\sqrt{3} \cdot F$.

5. (1) Let O be the centre of the circle described about the triangle ABC.

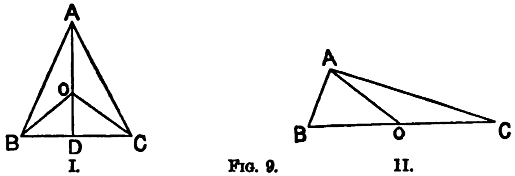

FIG. 9.

Then $OA = OB = OC$.

Now AO passes through D, the middle point of BC, by the conditions of the question. (See Examples—II. 3 (1).)

Hence (fig. I.) OD is perpendicular to BC (EUCLID, III. III.); $\therefore AB = AC$ (EUCLID, I. IV.), and therefore ABC is isosceles.

If O, the centre of the circle, lies in BC, then $OA = OB = OC$, and the angle BAC, being the angle in a semicircle, is a right angle; that is, the triangle is right-angled.

(2) The two smaller forces being at right angles, let the measures of the three forces, taken in order of magnitude and represented by AB, AC, AD be $x-d$, x, $x+d$.

Then $AE^2 = AC^2 + CE^2$,

and $\therefore (x+d)^2 = x^2 + (x-d)^2$;

or, $x^2 + 2dx + d^2 = x^2 + x^2 - 2dx + d^2$;

$\therefore 4dx = x^2$.

Hence $x = 4d$, and the measures of the forces are $3d$, $4d$, $5d$; that is, the common difference d is one-third of the least force.

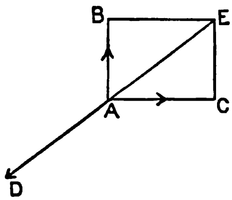

Fig. 10.

N.B.—There is an error in this Example in the edition of 1875: for *greatest* read *least but one.*

6. This is worked in the same way as I. 9, and we get

$$x^2 = 4 + 9 + 2 \,.\, 2 \,.\, \frac{3}{2} = 13 + 6 = 19.$$

7. This is worked as I. 9, observing that the resultant is given, and that x must be put for one of the components; then

$$3^2 = 2^2 + x^2 + 2 \,.\, 2 \,.\, \frac{x}{2};$$

$$\text{or, } 9 = 4 + x^2 + 2x.$$

Hence $x^2 + 2x = 5$; whence $x = \sqrt{6} - 1$.

8. Let O be the point of application, OE the line pointing to the East.

Let OA, OB, represent the components, so that $\angle AOB = 135°$.

Draw AR at right angles to OE.

Then since $\angle AOR = 45°$, and $\angle ARO = 90°$, $\angle OAR = 45°$.

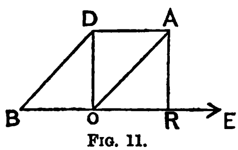

Fig. 11.

$\therefore$ if the measure of AO be $\sqrt{2} \,.\, P$, the measure of $OR = P$, and since $DA = BO$, the measure of $DA = P$.

Hence $DORA$ is a parallelogram.

$\therefore$ OD is parallel to AR, and is therefore in the direction of the North, and $OD = AR$, and therefore its magnitude is P.

9. Let OA, OB, OC represent the three forces P, $2P$, $\sqrt{3}.P$ acting at O.

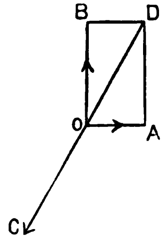

Fig. 12.

Complete the parallelogram $OADB$.

Then since OD represents $2P$, and

$(2P)^2=(\sqrt{3}.P)^2+P^2$, $\angle DAO$ is a right angle;

$\therefore$ since $OD=2.OA$,

$\angle DOA=60°$; and $\therefore$ $\angle BOD=30°$.

Hence $\angle AOB=90°$,

$\angle BOC=150°$,

$\angle AOC=120°$.

10. Produce RO to any point D, and draw DB parallel to OP, meeting OQ in B.

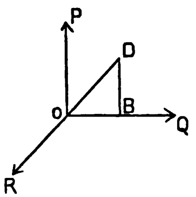

Fig. 13.

Then the sides of the triangle DOB being parallel to the forces P, Q, R, are proportional to them.

Now since $\angle ROQ=135°$, $\therefore$ $\angle DOB=45°$.

Also $\angle DBO=90°$.

$$\therefore P:Q:R=DB:BO:DO$$
$$=1\ :1\ :\sqrt{2}.$$

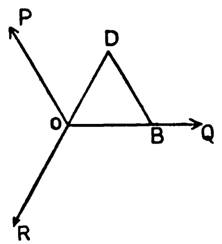

Fig. 14.

11. Construct a diagram as in Example 10.

Then since $P=Q=R$,

$$DB=BO=OD.$$

$\therefore$ $\angle DOB=60°$;

$\therefore$ $\angle ROQ=120°$.

Similarly it may be shown that

$$\angle POQ=\angle POR=120°.$$

12. Let AB, AC be the components, AD the resultant perpendicular to AB.

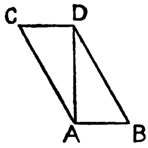

FIG. 15.

Now, since $\angle BAC=120°$, $\therefore \angle ABD=60°$.

Hence in triangle DAB

$$AB:BD:DA=1:2:\sqrt{3};$$

$$\therefore AB:AC:AD=1:2:\sqrt{3}.$$

13. Let AB, AC represent the components, $AC=2AB$, and AD the resultant.

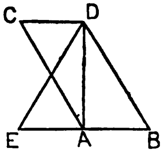

FIG. 16.

Produce BA to E, making $AE=AB$.

Then since $\angle CAB=120°$, $\therefore \angle DBE=60°$; and $\therefore$, since $BE=BD$,

$$\angle BDE=\angle BED=60°.$$

Hence DBE is an equilateral triangle, and DA, which bisects the base, is at right angles to BA.

$$\therefore AD:AB=\sqrt{3}:1, \text{ or, } AD=\sqrt{3}.P.$$

14. Let AB, AC be the components, and let AD the resultant $=2.AB$.

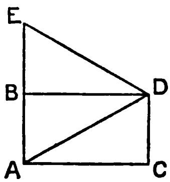

FIG. 17.

Produce AB to E, making $BE=AB$.

Then $AE=AD$.

But, since DB bisects AE at right angles,

$$AD=DE;$$

$\therefore AED$ is an equilateral triangle,

and $\therefore \angle BAD=60°$.

15. Let AB, AC be the components, AD the resultant$=AC$.

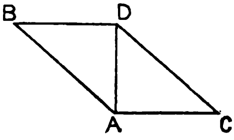

FIG. 18.

Then since $\angle BAC=135°$, $\therefore \angle DCA=45°$.

Hence $\angle ADC=\angle DCA=45°$,

and $\therefore \angle DAC=90°$.

Then $AB:AC=CD:AC=\sqrt{2}:1$.

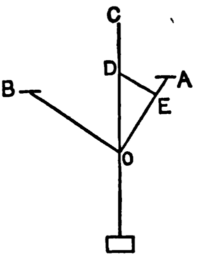

FIG. 19.

16. Let AO, BO be the strings,

CO the vertical line

$\angle AOC = 30°$.

From any point D in OC draw DE parallel to OB.

Then the sides of the triangle DOE are parallel to the three forces.

Now $\angle DOE = 30°$, $\angle DEO = 90°$,

and $\therefore \angle EDO = 60°$,

$\therefore$ tension of AO : tension of $BO = EO : ED = \sqrt{3} : 1$.

17. (1) Construct a diagram as in Example 13, and AD the resultant, being at right angles to AB, makes an angle of 30° with AC.

(2) The components being equal, the resultant bisects the angle between them, and this being an angle of 135°, the resultant makes an angle of $67\frac{1}{2}°$ with each component.

18. Produce AD to E, making $DE = AD$.

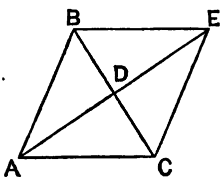

FIG. 20.

Join BE, CE.

Then $\because AD = ED$, and $BD = CD$,

and $\angle BDE = \angle ADC$;

$\therefore \angle BED = \angle DAC$.

Hence BE is parallel to AC.

Again, $\because AD = ED$, and $BD = CD$, and $\angle ADB = EDC$,

$\therefore \angle DBA = \angle DCE$.

Hence AB is parallel to EC.

Thus $ABEC$ is a parallelogram, and AE is the resultant of AB, AC.

19. Construct a diagram as in Example 10.

Let R, P, Q be the order of magnitude, R being the greatest.

Then OD, DB, BO are in descending order;

$\therefore$ $\angle OBD$, $\angle BOD$, $\angle ODB$ are in descending order.

$\therefore$ $\angle POQ$, $\angle QOR$, $\angle ROP$ are in *ascending* order,

for $\angle OBD + \angle POQ = 2$ right angles, and similarly for the others.

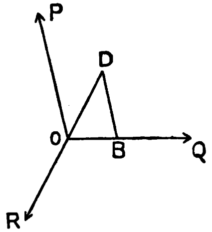

FIG. 21.

20. Take a parallelogram $ABCD$, having $\angle BAD$ less than $\angle ABC$.

Now ABD and BAC are two triangles having two sides equal, each to each, but the included $\angle BAD$ in the one less than the included $\angle ABC$ in the other,

$\therefore$ BD is less than AC.

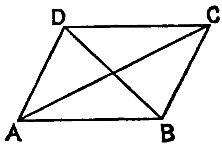

FIG. 22.

Or, by Trigonometry,

$$R^2 = P^2 + Q^2 + 2PQ \,.\, \cos\alpha\,;$$

and as α increases from $0°$ to $90°$, $\cos\alpha$ diminishes,

and as α increases from $90°$ to $180°$, $\cos\alpha$ increases, but is negative;

$\therefore$ R is always decreased as α increases.

21. Let α be the angle between the strings, t the tension of each string, w the weight supported.

$$\text{Then } w^2 = t^2 + t^2 + 2t^2 \,.\, \cos\alpha.$$

Now, as the string is lengthened α is decreased,

$\therefore$ $\cos\alpha$ is increased;

and $\therefore$, since w remains constant, t is decreased.

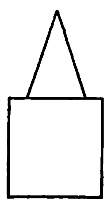
FIG. 23.

The pressure on the nail will not be affected by the length of the string.

22. Let A be the peg, and let AB, AC represent the equal forces P and P pulling at the ends of the string.

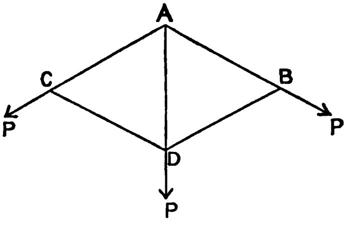

FIG. 24.

Then the strain on the peg is a force represented by AD, the diagonal of the rhombus $ACDB$.

Also, $AD=AB=AC$.

$\therefore$ ACD and ABD are equilateral triangles;

$\therefore$ $\angle CAD = \angle BAD = 60°$;

$\therefore$ $\angle CAB = 120°$.

23. The polygon, the sides of which represent the forces in magnitude and direction, must be a rhombus or a square, for all the sides are to be equal. Hence, since the opposite sides of the polygon are parallel, the forces must be opposite, two and two.

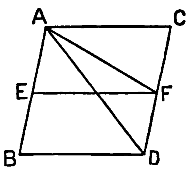

FIG. 25.

24. Take F, the middle point of CD, and join EF; then EF is parallel to AC.

Now, resultant of AD, $AC = 2 . AF$ (Example 18), and resultant of AE, $AC = AF$;

$\therefore$ resultant of AD, AC = twice resultant of AE, AC.

25. In the first system R is the resultant of P and Q; in the second system R' is the resultant of P and Q when these forces are reversed.

$\therefore$ R is equal and opposite to R'.

26. Take the diagram on page 30 of the *Statics*.

The force representing the combined effect of AB, BC, CD, DE is AE, and therefore the forces represented by AB, BC, CD, DE are kept in equilibrium by a force represented by a line equal and opposite to AE.

Hence X lies in EA produced, so that $AX=AE$.

27. Let O be the point *within* the quadrilateral $ABCD$.

Now E, F, G, H, being the points of bisection of the sides of the quadrilateral, being joined would form a parallelogram, and the diagonals of this parallelogram bisect each other in N.

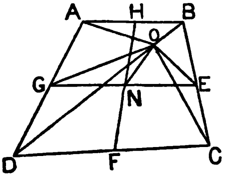

FIG. 26.

Then resultant of OA, OD is $2 . OG$,

and resultant of OB, OC is $2 . OE$;

$\therefore$ resultant of OA, OB, OC, OD is $2 .$ resultant of OG, OE.

But resultant of OG, OE is $2 . ON$;

$\therefore$ resultant of OA, OB, OC, OD is $4 . ON$.

The construction and proof are precisely the same when O is *without* the quadrilateral.

EXAMPLES—III. (p. 40).

1. Let AB represent a force of 12 lbs., AC, AD the components.

Then $\angle CAB=90°$, $\angle ABC=30°$;

and $\therefore$ $\angle ACB=60°$.

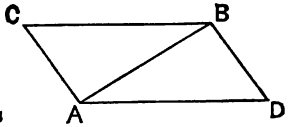

FIG. 27.

Then $AC : CB : AB = 1 : 2 : \sqrt{3}$;

$\therefore$ $AC : 12 = 1 : \sqrt{3}$, or, AC represents a force of $4\sqrt{3}$ lbs;

and $CB : 12 = 2 : \sqrt{3}$, or, AD represents a force of $8\sqrt{3}$ lbs.

2. (1) Let AB represent a force of 10 lbs., AD, AC the components, each making an angle of 30° with AB, and being therefore equal.

Draw BE at right angles to AC produced.

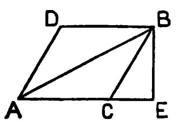

FIG. 28.

Then $\angle BCE = \angle DAC = 60°$;

and $\therefore$ $BC = 2CE$.

Now $AB^2 = AC^2 + CB^2 + 2AC . CE$,

$$\text{or } 100 = AC^2 + AC^2 + 2AC . \frac{AC}{2};$$

$$\therefore 100 = 3AC^2, \text{ and } \therefore AC = \frac{10\sqrt{3}}{3} = AD.$$

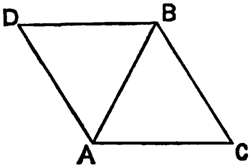

FIG. 29.

(2) Since $\angle BAD = \angle BAC = 60°$,

$$\therefore \angle ACB = 60°;$$

$$\therefore AC = AD = AB = 10 \text{ lbs.}$$

3. Let a be the angle.

Then, since $R^2 = P^2 + Q^2 + 2PQ \cdot \cos a$

$$9^2 = 9^2 + 6^2 + 2 \cdot 9 \cdot 6 \cdot \cos a,$$

$$\text{or } 81 = 81 + 36 + 108 \cos a;$$

$$\therefore \cos a = -\frac{36}{108} = -\frac{1}{3}.$$

4. (1) Let AB represent the given force, and let CD, EF represent the other forces.

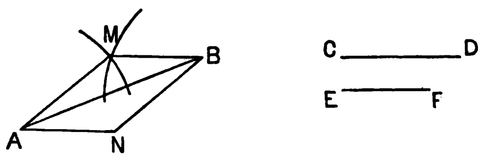

FIG. 30.

With centre A, and distance CD, describe a circle.
With centre B, and distance EF, describe a circle.
Let M be a point where the circles intersect.
Join AM, MB, and complete the parallelogram $AMBN$.
Then AM, AN represent the component forces.

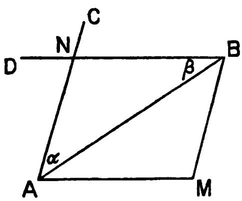

FIG. 31.

(2) Let a and β be the angles that the component forces are to make with the given force represented by AB.

Draw AC making $\angle BAC = a$,
and BD making $\angle DBA = \beta$,

and let N be the intersection of AC, BD.
Complete the parallelogram $AMBN$.

Then AN, AM represent the component forces.

5. Complete the parallelogram $ACDB$.

O is the middle point of AD and BC.

$AC = 13$, $OC = 5$, and AOC is a right angle, because BAC is an isosceles triangle.

Then $AO^2 = 13^2 - 5^2 = 144$;

$\therefore AO = 12$, and $\therefore AD = 24$.

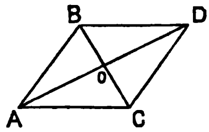

FIG. 32.

6. First, C lies on the circumference of the circle described with A as centre and AB as radius.

Next, D lies on the circumference of the circle described with B as centre and BA as radius.

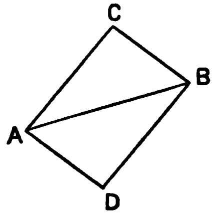

FIG. 33.

7. If P and Q be the component forces and a the angle between their directions, and if Q be reversed, the angle that it makes with P is $180° - a$. Hence, if R and S be the resultants,

$$R^2 = P^2 + Q^2 + 2PQ \,.\, \cos a,$$
$$S^2 = P^2 + Q^2 + 2PQ \,.\, \cos(180° - a)$$
$$= P^2 + Q^2 - 2PQ \,.\, \cos a;$$
$$\therefore R^2 + S^2 = 2(P^2 + Q^2), \text{ which is independent of } a.$$

8. Let T be the tension of each string in the first case, t the tension of each string in the second case, W the weight of the picture.

$$\text{Then } W^2 = T^2 + T^2 + 2T^2 \,.\, \cos 60°,$$
$$\text{and } W^2 = t^2 + t^2 + 2t^2 \,.\, \cos 120°;$$
$$\therefore 2T^2 + 2T^2 \,.\, \tfrac{1}{2} = 2t^2 + 2t^2 \,.\, \times(-\tfrac{1}{2}),$$
$$\text{or, } 3T^2 = t^2;$$
$$\therefore T^2 : t^2 = 1 : 3;$$
$$\therefore T : t = 1 : \sqrt{3}.$$

9. Let AC represent the force of 9 lbs. Then take the formula for finding the cosine of one of the angles of a triangle in terms of the sides (TRIGONOMETRY, Art. 179),

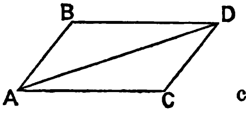

FIG. 34.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \text{ and we have}$$

$$\cos DAC = \frac{144 + 81 - 36}{2 \times 12 \times 9} = \frac{225 - 36}{216} = \frac{189}{216} = \frac{7}{8}$$

$$\cos BAD = \frac{144 + 36 - 81}{2 \times 12 \times 6} = \frac{180 - 81}{144} = \frac{99}{144} = \frac{11}{16}.$$

10. Let P_1 and P_2 be any two positions of P, and let C be the point in the arc ACB through which the resultant passes when P is in the position P_1. Join CP_2.

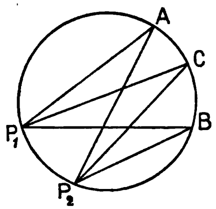

FIG. 35.

Then $\angle AP_1B = \angle AP_2B$, and $\angle AP_1C = \angle AP_2C$. (EUCLID, III. XXI.)

Now since the two forces are the same at P_1 as at P_2, and act at the same angle, the resultant must make with them the same angle in both positions.

$\therefore P_2C$ must be the direction of the resultant for the position P_2.

11. Let the string lie on the quadrant PCQ, and let O be the centre of the circle, and OC a vertical line.

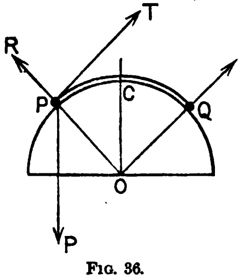

FIG. 36.

Then the forces acting on P are its weight P, R the pressure of the semicircle acting along the normal OR, and T the tension of the string, which acts as a tangent at P. Let $\angle COP = \alpha$.

Then $T : P = \sin\alpha : \sin 90^\circ$. (Art. XXXIX.)

$$\therefore T = P \,.\, \sin\alpha.$$

Similarly $T = Q \,.\, \sin COQ = Q \,.\, \cos\alpha$.

$$\therefore P \,.\, \sin\alpha = Q \,.\, \cos\alpha, \text{ and } \therefore \tan\alpha = \frac{Q}{P}.$$

12. Resultant of AE, BE is $2FE$.
" " DF, CF is $2EF$.

Resultant of BG, CG is $2HG$.
" " AH, DH is $2GH$.

Hence the eight forces clearly form a system in equilibrium.

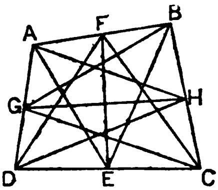

FIG. 37.

13. From Art. XLVIII.

$$R^2=(8+4\cos 30^\circ+4\cos 60^\circ)^2,$$
$$+(6+4\cos 60^\circ+4\cos 30^\circ)^2$$
$$=(8+2\sqrt{3}+2)^2+(6+2+2\sqrt{3})^2$$
$$=(10+2\sqrt{3})^2+(8+2\sqrt{3})^2$$
$$=100+40\sqrt{3}+12+64+32\sqrt{3}+12$$
$$=188+72\sqrt{3}=4\ (47+18\sqrt{3});$$
$$\therefore R=2\sqrt{(47+18\sqrt{3})}.$$

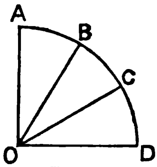

FIG. 38.

Also $\tan\theta=\dfrac{8+2\sqrt{3}}{10+2\sqrt{3}}=\dfrac{4+\sqrt{3}}{5+\sqrt{3}}=\dfrac{(4+\sqrt{3})(5-\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})}=\dfrac{17+\sqrt{3}}{22}.$

14. Let P be the given point.

Take M and N the middle points of AB and CD. The line joining M and N passes through O, the centre.

Resultant of $PA, PB=2\,.\,PN$.
Resultant of $PC, PD=2PM$;
$\therefore$ Resultant of PA, PB, PC, PD
$=2$(resultant of PM, PN)
$=2\times 2\,.\,PO$.
$=4PO$.

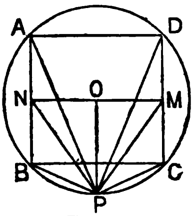

FIG. 39.

15. PM represents joint effect of PO, OM.

NQ represents joint effect of OQ, NO;
$\therefore PM, NQ$ represent joint effect of PQ, NM.
" " of AD, AB,
" " of AC.
$\therefore$ PM, NQ, CA form a system in equilibrium.

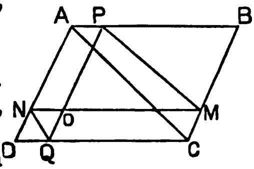

FIG. 40.

EXAMPLES—IV. (p. 45).

(1) $20 : 30 = BC : 5$, or, $100 = 30BC$, $\therefore BC = 3\frac{1}{3}$ inches.

(2) Here $P + Q = 28$, and $\therefore P : 28 - P = 3 : 4$;

$\therefore 4P = 84 - 3P$, whence $P = 12$, and $\therefore Q = 16$.

Then $12 : 16 = 7 - AC : AC$, whence $AC = 4$ inches.

(3) $P = R - Q = (14\frac{1}{2} - \frac{1}{2})$ lbs. $= 14$ lbs.

Then $14 : \frac{1}{2} = BC : 2$, and $\therefore BC = 56$ inches.

Hence $AB = 58$ inches $= 4$ feet 10 inches.

(4) $3 : 5 = AC - 12 : AC$, and $\therefore AC = 30$ inches $= 2$ feet 6 inches.

(5) $10 : Q = 18 - 6 : 18$, and $\therefore Q = 15$ lbs.

EXAMPLES—V. (p. 52).

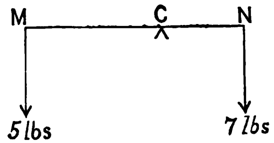

FIG. 41.

1. $5 : 7 = CN : CM$

$= CN : 6 - CN$;

$\therefore 30 - 5CN = 7CN$;

$\therefore CN = 2\frac{1}{2}$ feet, and $CM = 3\frac{1}{2}$ feet.

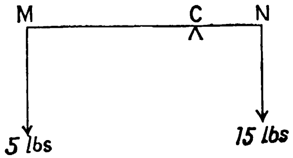

FIG. 42.

2. $CM : CN = 15 : 5$

$= 3 : 1$.

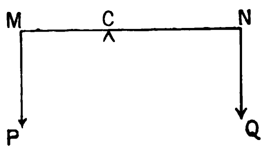

FIG. 43.

3. $P : \quad = CN : CM$,

and $P + Q = 99$;

$\therefore P : 99 - P = 7 : 4$,

$\therefore 4P = 693 - 7P$;

$\therefore P = 63$ lbs., and $\therefore Q = 36$ lbs.

4. $Q=(16-12)$ lbs. $=4$ lbs.;

$\therefore 12:4=CN:1$ foot;

$\therefore CN=3$ feet.

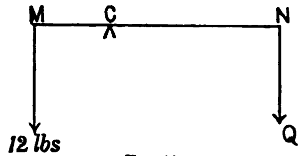

FIG. 44.

5. $CN=36-CM$;

$\therefore 5:7=36-CM:CM$;

$\therefore 5CM=252-7CM$;

$\therefore CM=21$ inches.

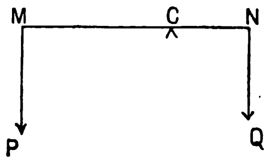

FIG. 45.

6. $CM:CN=5:2$, the condition that one of the arms is 5 inches longer than the other having no effect on their *relative* value.

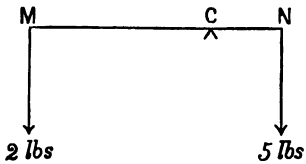

FIG. 46.

7. Let P be the smaller weight.

$$\text{Then } \frac{P}{Q}=\frac{CN}{CM}.$$

Let x be added to P and taken from Q, and let $P+x$ act at N, and $Q-x$ at M.

$$\text{Then } \frac{Q-x}{P+x}=\frac{CN}{CM}.$$

$$\text{Hence } \frac{P}{Q}=\frac{Q-x}{P+x};$$

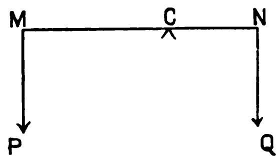

FIG. 47.

or, $P^2+Px=Q^2-Qx$,

or $(P+Q)x=Q^2-P^2$, and $\therefore x=Q-P$.

If we take P the larger weight, $x=P-Q$.

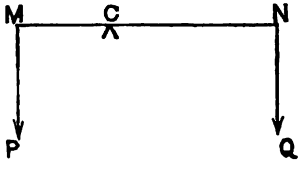

FIG. 48.

8. $CN = 33 - CM$;

$\therefore CM : 33 - CM = 3 : 8$;

$\therefore 8CM = 99 - 3CM$;

$\therefore CM = 9$ inches.

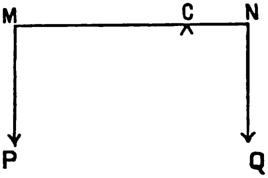

FIG. 49.

9. $CM : 8 - CM = 3 : 1$;

$\therefore CM = 24 - 3CM$;

$\therefore CM = 6$ feet.

10. Let C be the first position of the fulcrum.

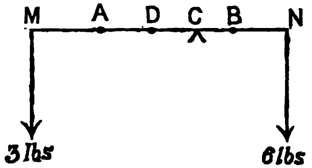

FIG. 50.

Then $CM : 12 - CM = 6 : 3$;

$\therefore 3CM = 72 - 6CM$;

$\therefore CM = 8$ feet.

Now move the weights of 3 lbs. and 6 lbs. to A and B, and let D be the new position of the fulcrum. Then $AB = 8$ feet, and $DB = 8 - AD$, and $AD : 8 - AD = 6 : 3$;

$\therefore 3AD = 48 - 6AD$, or $AD = 5\frac{1}{3}$ feet.

Now $AC = 6$ feet, $\therefore DC = \frac{2}{3}$ feet $= 8$ inches.

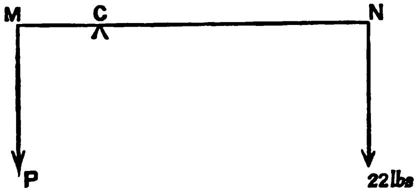

FIG. 51.

11. $CM = 8$in., $CN = 38$in.

$\therefore P : 22 = 38 : 8$;

$\therefore 8P = 836$;

$\therefore P = 104\frac{1}{2}$ lbs.

12. Let x be the length of AC in inches.

Then $CN : CM = 10 : 15$,

or, $\frac{x}{2} : x - 6 = 2 : 3$;

$\therefore \frac{3x}{2} = 2x - 12$,

or, $x = 24$ inches $= 2$ feet.

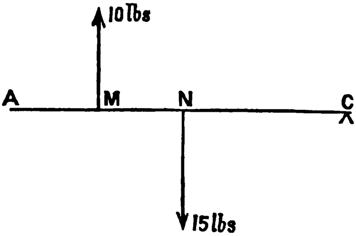

FIG. 52.

13. $CN : CM = 14 : 25$;

$\therefore CN : 36 = 14 : 25$;

$\therefore 25 CN = 504$;

$\therefore CN = 20\frac{4}{25}$ inches.

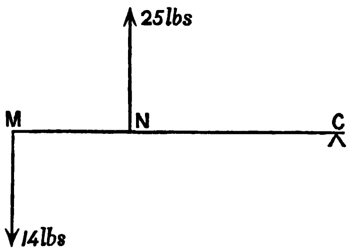

FIG. 53.

14. Take N the middle point of AB, this point being equidistant from C and D: a weight of 24 lbs. acting at N will produce the same effect as that which is produced by the two weights acting at C and D, and it may therefore be substituted for them.

Then, if x be the force acting upwards at A,

$$x : 24 = BN : AB$$
$$= 1 : 2;$$

$\therefore 2x = 24$, $\therefore x = 12$ lbs.

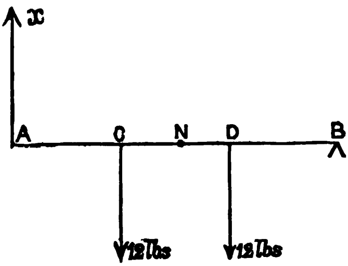

FIG. 54.

15. (1) Draw ON at right angles to BQ.

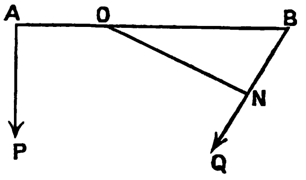

FIG. 55.

Then $ON : OB = \sqrt{3} : 2$;

$\therefore ON = \frac{5\sqrt{3}}{2}$.

And $P : Q = ON : OA$,

or $4 : Q = \frac{5\sqrt{3}}{2} : 3$;

$\therefore 24 = 5\sqrt{3} \,.\, Q$, or $Q = \frac{24}{5\sqrt{3}} = \frac{24\sqrt{3}}{15}$

$= \frac{8\sqrt{3}}{5}$ lbs.

(2) Draw OM, ON at right angles to AP, BQ.

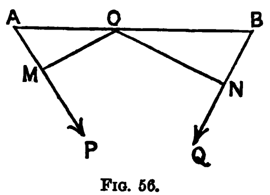

FIG. 56.

Then since triangles AOM, BON are similar,

$ON : OM = OB : OA$

$= 5 : 3$.

Now $P : Q = ON : OM$;

$\therefore 3 : Q = 5 : 3$, and $\therefore Q = 1\frac{4}{5}$ lbs.

(3) Draw OM, ON at right angles to AP, BQ.

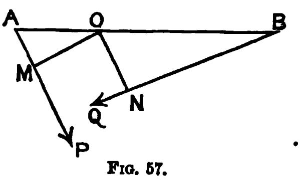

FIG. 57.

Then $OM = \frac{AO}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

$ON = \frac{OB}{2} = \frac{5}{2}$;

$\therefore 2 : Q = \frac{5}{2} : \frac{3}{\sqrt{2}}$, or $\frac{2 \times 3}{\sqrt{2}} = \frac{5 \times Q}{2}$;

$\therefore Q = \frac{2 \times 3 \times 2}{5 \times \sqrt{2}} = \frac{2 \times 3 \times 2 \times \sqrt{2}}{5 \times 2} = \frac{6\sqrt{2}}{5}$ lbs.

Examples—VI. (p. 66).

1. The weight of the rod AB may be concentrated at N its middle point.

Then if C be the centre of gravity

$$AC = \frac{8}{9} \text{ of } AN = \frac{8 \times 18}{9} \text{ in.} = 16 \text{ in.}$$

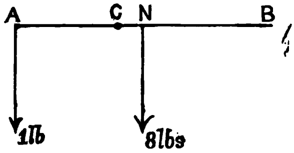

Fig. 58.

2. The weight of 4 lbs. at A and 2 lbs. at N, the middle point of AB, are equivalent to a weight of 6 lbs. at D, AD being one-third of AN, or 2 inches.

$\therefore DB = 10$ inches, and the centre of gravity of 6 lbs. at D and 1 lb. at B is $\frac{1}{7}$ of 10 inches, or $1\frac{3}{7}$ inches from D.

Hence the centre of gravity of the system is $3\frac{3}{7}$ inches from A.

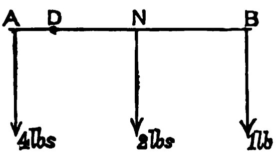

Fig. 59.

3. By Art. 71, if N be the centre of gravity, and D be taken a the fixed point whose distance from N we want to find in inches.

$$ND = \frac{P.AD + P.BD + P.CD}{P + P + P + P}$$

$$= \frac{6P + 5P + 3P}{4P} = \frac{14}{4} = 3\tfrac{1}{2}.$$

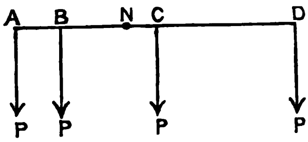

Fig. 60.

4. To find the distance of N the centre of gravity from A in feet.

$$AN = \frac{8 \,.\, AB + 4AC + 2AD + 1 \,.\, AE}{16 + 8 + 4 + 2 + 1}$$

$$= \frac{8 + 8 + 6 + 4}{31} = \frac{26}{31} \text{ feet.}$$

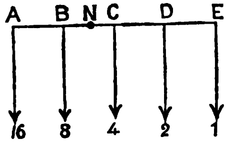

Fig. 61.

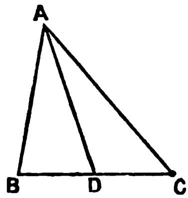

FIG. 62.

5. The centre of gravity of B and C lies in BC, and it also lies in the line AD passing through the centre of gravity of the triangle.

Hence it lies at D, the middle point of BC.

$\therefore B=C$, and similarly it may be shown that $A=B=C$.

6. Let BC be the base, ABC, DBC any two of the triangles. Draw AM, DM to M the middle point of BC.

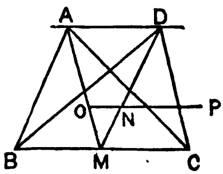

FIG. 63.

Let O be the centre of gravity of ABC, and draw OP parallel to AD, cutting DM in N.

Then shall N be the centre of gravity of DBC.

For since ON is parallel to AD,

$DN:NM=AO:OM$ (EUCLID, VI. II.)

$=2:1$.

7. DE, joining the middle points of AB, AC, is parallel to BC.

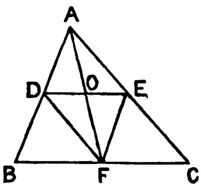

FIG. 64.

Hence AF bisects DE in O.

$\therefore$ the centre of gravity of DFE lies in FO, that is, in AF, which passes through the centre of gravity of ABC. Also, the distance of the centre of gravity of DFE from F is $\frac{2}{3}.FO=\frac{1}{3}.AF$, which is the distance of the centre of gravity of ABC from F. Thus the triangles have the same centre of gravity.

8. Taking the diagram of Example 6, suppose O and N to be the centres of gravity of two of the triangles.

Then since $AO=2OM$, and $DN=2NM$,

$$\therefore AO:OM=DN:NM;$$

$$\therefore AD \text{ is parallel to } ON.$$

9. Let P, Q be two equal particles; P', Q', the same particles in new positions.

Now, since $PP'=QQ'$, and the angles of the triangles $PP'O$, $QQ'O$ are equal, each to each,

$\therefore PO=QO$, and $P'O=Q'O$;

$\therefore$ O is the centre of gravity of the particles in both positions.

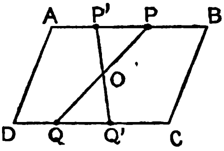

FIG. 65.

10. Let D be the centre of gravity of A and B, and E that of A and C. Let CD, BE intersect in O.

Then the centre of gravity of A, B, C lies in CD and also in BE.

$\therefore$ O is the centre of gravity of A, B, C.

Hence N, the point in BC where AO meets BC, must be the centre of gravity of B and C.

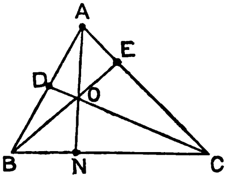

FIG. 66.

11. Let O be the centre of gravity of the equilateral triangle ABC. Then AD bisects BC at right angles.

Hence $OB=OC$;

and similarly it may be shown that $OA=OC$.

$\therefore$ O is the centre of the circle.

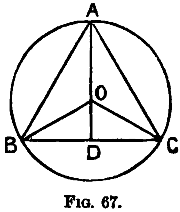

FIG. 67.

12. Taking the diagram of Example 11,

$BD=CD$, because O is the centre of gravity of the triangle,

and $OB=OC$, because O is the centre of the circle.

$\therefore \angle ODB = \angle ODC =$ a right angle.

$\therefore AB=AC$, and similarly it may be shown that $AB=BC$.

$\therefore ABC$ is equilateral.

13. To keep the line, passing vertically through the common centre of gravity of his own body and the weight, within the base covered by his feet.

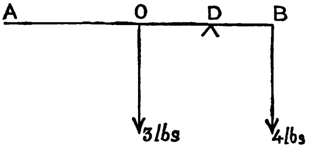

FIG. 68.

14. Let AB be the rod, O its middle point, D the edge of the table.

Then we have 3 lbs. acting at O, and 4 lbs. at B.

Hence, since $OB=7$ inches,

$OD=4$ inches;

and $\therefore AD=11$ inches.

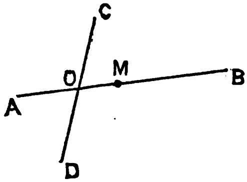

FIG. 69.

15. Let the weight of each particle be P.

Then P at A and P at B are equivalent to $2P$ at M, the middle point of AB.

Also, P at C and P at D are equivalent to $2P$ at O, the middle point at CD.

$\therefore$ the centre of gravity lies midway between O and M.

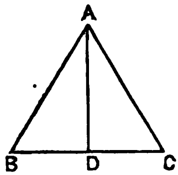

FIG. 70.

16. Let A be the point of suspension.

Then the vertical line AD passes through the middle point of BC, and since BC is horizontal, the angles at D are right angles.

Hence $AB=AC$. (EUCLID, I. IV.)

17. Let O, B be the centres of gravity, A the middle point of the common side, C the centre of gravity of the two.

OA will be perpendicular to the side of the square.

Draw BD at right angles to the same side.

Let a be the side of the square, b the altitude of the triangle.

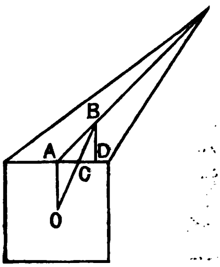

FIG. 71.

Then area of triangle $=\frac{ab}{2}$.

Area of square $=a^2$,

$$\therefore \frac{ab}{2} : a^2 = OC : BC$$

$$= OA : BD \text{ (by similar triangles)}$$

$$= \frac{a}{2} : \frac{b}{3};$$

$$\therefore \frac{ab^2}{6} = \frac{a^3}{2}, \text{ and } \therefore b = \sqrt{3}\,.\,a.$$

18. ABC, $A'B'C'$ are the two positions of the triangle, $CDB'D'$ being a vertical line bisecting AB and $C'A'$.

Let $\angle DCA=\theta$, $AC=x$, $AB=y$.

Then $\angle CAD=\theta$, and $\angle ADB'=2\theta$,

$\therefore \angle A'B'D'=\angle AB'D=90^\circ-2\theta$,

and $\angle B'A'D'=\angle BAC=\theta$;

$\therefore \angle A'D'B'=180^\circ-(90^\circ-2\theta)-\theta=90^\circ+\theta$.

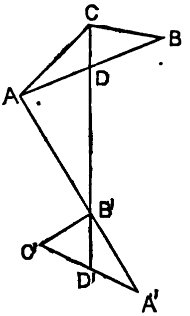

FIG. 72.

Then $\dfrac{\sin(90^\circ+\theta)}{\sin(90^\circ-2\theta)}=\dfrac{A'B'}{A'D'}=\dfrac{2y}{x}$;

$$\text{or, } \frac{\cos\theta}{\cos 2\theta}=\frac{2y}{x}, \text{ or, } \frac{\frac{x}{y}}{\frac{2x^2}{y^2}-1}=\frac{2y}{x};$$

$$\text{or, } \frac{xy}{2x^2-y^2}=\frac{2y}{x};$$

$$\therefore x^2=4x^2-2y^2;$$

$$\therefore 2y^2=3x^2;$$

$$\therefore x:y=\sqrt{2}:\sqrt{3};$$

$$\therefore AC:CB=\sqrt{2}:1.$$

19. Place 3 lbs. at A, 4 lbs. at B, and 5 lbs. at C.

These are equivalent to 3 lbs. at A, B, C, together with 1 lb. at B, and 2 lbs. at C.

These are equivalent to 9 lbs. at O, the centre of gravity of the triangle, together with 3 lbs. at D, one-third of the distance of C from B.

Hence the centre of gravity is one-fourth of the distance of D from O, measured from O.

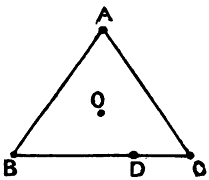

FIG. 73.

20. Place $2P$ at B, $2P$ at C, and P at A.

These are equivalent to $4P$ at D, the middle point of BC, and P at A.

Hence the centre of gravity lies in AD at a distance from D one-fifth of AD.

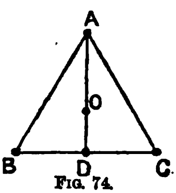

FIG. 74.

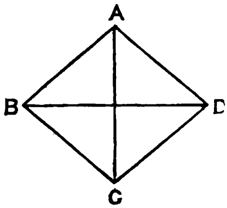

FIG. 75.

21. Let A be the point of suspension.

The centre of gravity of the parallelogram being the point of intersection of the diagonals, AC is a vertical line, and since the diagonals of a rhombus intersect at right angles;

$\therefore$ BD is horizontal.

22. Draw MON, ROS through O, the point of intersection of the diagonals, parallel to the sides of the original parallelogram.

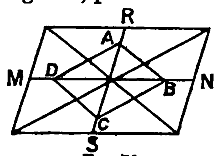

FIG. 76.

Then MON, ROS will bisect the sides.

Hence the centres of gravity, A, B, C, D, of the four triangles into which the parallelogram is divided by its diagonals (all of which triangles are equal), are equidistant from O. Now the lines joining the extremities of the straight lines AC, BD, which bisect each other, form a parallelogram, as can be easily shown.

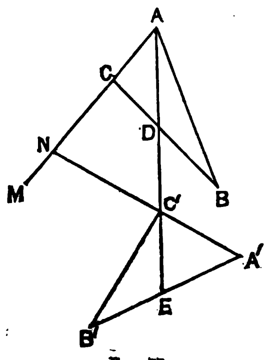

FIG. 77.

23. Let $ADC'E$ be a vertical line.

$$\angle MNC' = \angle NAC' + AC'N$$
$$= \angle CAD + \angle A'C'E.$$

Now since AD bisects BC;

$$\therefore DC = \tfrac{1}{2}BC = AC;$$
$$\therefore \angle CAD = 45^\circ.$$

And since $C'E$ bisects $A'B'$, E is the centre of the circle described round $A'B'C'$;

$$\therefore EC' = EA'.$$

And $\therefore$ $\angle A'C'E = \angle$ at $A' = \angle A$;

$$\therefore \angle MNC' = 45^\circ + A.$$

24. Let C be the right angle, CD the vertical line, bisecting AB in D.

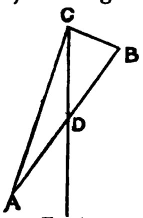

Fig. 78.

Then $\angle CDB = \angle DCA + \angle CAD$

$= 2\angle DAC$ (for $DA = DB = DC$).

But $\angle DAC + \angle CBA = 90°$;

or, $\angle DAC + 5\angle DAC = 90°$,

and $\therefore\ \angle DAC = 15°$;

$\therefore\ \angle CDB = 30°$.

25. First remark that under the conditions of the question, if AB, AC be the equal sides of the triangle, and O the centre of gravity, $\angle OBC = \angle OCB = 45°$, since $OD = BD = DC$.

Next, let $BMC'N$ be the vertical line passing through the points of suspension, and let $B'C'$ produced meet BC, or BC produced, in R.

Then

$180° - \angle CRC' = \angle CBM + \angle RC'B$

$= \angle CBM + \angle B'C'N$

$= 45° + 45° = 90°$.

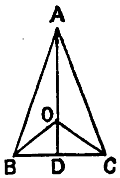

Fig. 79.

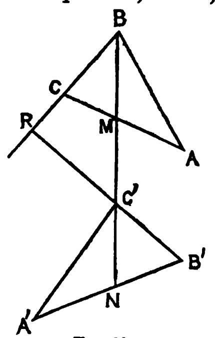

Fig. 80.

26. Since $AC = 2BC$, C is the centre of gravity of the weights acting at A and B.

$\therefore$ C is the centre of gravity of the three weights.

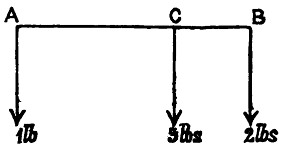

Fig. 81.

27. Let P be the weight of each of the particles acting at A, B, D.

Then P at B and P at D are equivalent to $2P$ at C.

Then if G be the centre of gravity of P at A, and $2P$ at C,

$$AG = \frac{2}{3} \text{ of } AC = \frac{4}{9} \text{ of } AD.$$

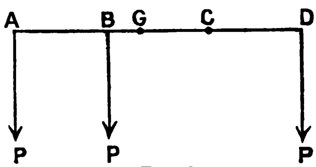

Fig. 82.

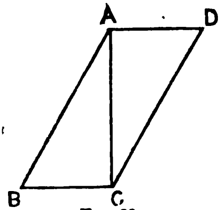

Fig. 83.

28. The limit in length of AB is attained when the diagonal AC is perpendicular to BC, for then the centre of gravity lies just over C.

In this case, since $\angle ABC = 60°$,

$$AB = 2BC = 12 \text{ inches.}$$

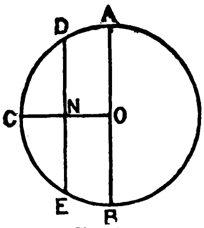

Fig. 84.

29. DE bisects OC in N.

Then the forces, each equal P, acting at O, A, B, C, D, E are equivalent to $2P$ at O, $2P$ at N, P at O, P at C, and therefore to $2P$ at O, and $4P$ at N.

Hence if G be the centre of gravity, it lies in OC, and is such that

$$OG = \frac{2}{3} \text{ of } ON = \frac{1}{3} \text{ of } OC.$$

30. The centre of gravity of the body coincides with the centre of gravity of the two parts taken together, and as this must be in the line joining G_1 and G_2, it follows that G must be in that line.

31. O is the centre of gravity of the square, G of the triangle, H of the figure $AECD$. Let $2a$ be the side of the square, and let $EB = x$.

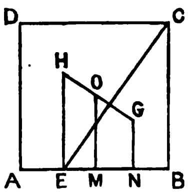

Fig. 85.

Then
$$4a^2 - ax : ax = OG : HO$$
$$= MN : EM$$
$$= a - \frac{2}{3} \cdot \frac{x}{2} : x - a;$$

$$\therefore\ 4a^2x - ax^2 - 4a^3 + a^2x = a^2x - \frac{ax^2}{3}, \text{ and hence}$$
$$x = 3a \pm a\sqrt{3}.$$

Thus $BE = a\sqrt{3}\,(\sqrt{3} - 1)$, and $AE = a\,(\sqrt{3} - 1)$, and $\therefore BE : AE = \sqrt{3} : 1$.

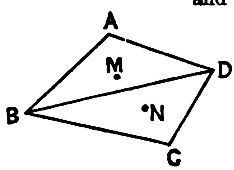

Fig. 86.

32. Let M, N be the centres of gravity of the equal triangles ABD, BCD.

Then since equal weights act at M and N, we may find the centre of gravity of the trapezium by taking the middle point of MN.

33. Let O be the centre of gravity of the square, N the centre of gravity of the triangle, $2a$ the length of a side of the square, x the distance of A from O.

Then area of square $=4a^2$,

area of triangle $=a\ (a+x)$,

and $AO : ON = a\ (a+x) : 4a^2 - a\ (a+x)$;

or, $x : a - \dfrac{a+x}{3} = a^2 + ax : 3a^2 - ax$;

or, $3x : 2a - x = a + x : 3a - x$;

or, $9ax - 3x^2 = 2a^2 + ax - x^2$.

$\therefore\ x^2 - 4ax = -a^2$,

and $x = 2a \pm \sqrt{3}a$.

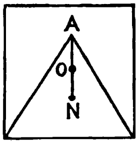

FIG. 87.

34. Let AB be the side on which the triangle will not stand, O the centre of gravity of the triangle, D the middle point of AC.

Now $\angle\ ABC$ is an obtuse angle,

$\therefore\ AC$ is the greatest side.

And $\because\ \angle\ CDB$ is an obtuse angle;

$\therefore\ CB$ is greater than CD;

$\therefore\ CB$ is greater than AB.

Also, $\because\ DBA$ is an obtuse angle;

$\therefore\ AD$ is greater than AB.

$\therefore\ AB$ is less than half of AC.

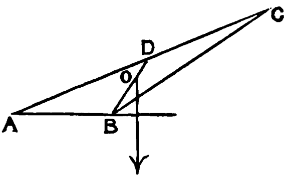

FIG. 88.

35. Draw DE, joining the middle points of AB, AC.

Then triangle ADE is one-fourth of triangle ABC.

Let R be the centre of gravity required, O the centre of gravity of triangle ADE, G the centre of gravity of triangle ABC.

Then $\dfrac{1}{4} \times OG = \dfrac{3}{4} \times RG$;

$\therefore\ OG = 3RG$,

$\therefore\ AG - AO = 3AR - 3AG$;

$\therefore\ \dfrac{2AF}{3} - \dfrac{2}{3} \cdot \dfrac{AF}{2} = 3AR - 2AF$;

$\therefore\ 3AR = \dfrac{7AF}{3}$, or, $AR = \dfrac{7}{9}$ of AF.

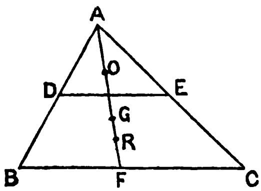

FIG. 89.

36. D, the middle point of BC, is equidistant from A, B, C, if BAC be a right angle.

$\therefore$, if O be the centre of gravity,

$AO = \dfrac{2}{3}$ of $AD = \dfrac{2}{3}$ of $BD = 2$ inches.

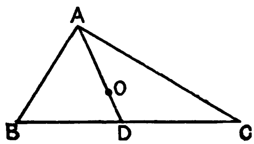

FIG. 90.

37. Let BQC be the equilateral triangle. The limit of equilibrium is reached when the centre of gravity of the two figures lies in the vertical line BC.

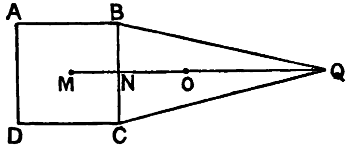

FIG. 91.

Let $2a$ be the side of the square, x the greatest height of the triangle, M, O the centres of gravity of the two figures, N the centre of gravity of the two together.

Then $4a^2 . MN = ax . NO$.

$$\therefore \; 4a^3 = ax \times \frac{x}{3}, \text{ or, } 12a^2 = x^2, \text{ and } \therefore \; x = 2\sqrt{3} . a.$$

38. Let O, E be the centres of gravity of the figures, G the common centre of gravity.

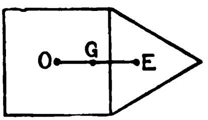

FIG. 92.

$$\text{Then } 4a^2 . OG = \sqrt{3} . a^2 . GE;$$
$$\text{or, } 4 . OG = \sqrt{3} . (OE - OG);$$
$$\text{or } (4 + \sqrt{3} .)OG = \sqrt{3} . \left(a + \frac{1}{3} . \sqrt{3} . a\right)$$
$$= a(\sqrt{3} + 1);$$
$$\therefore \; OG = \frac{a(\sqrt{3}+1)}{4+\sqrt{3}}.$$

39. The triangle ADE is one-third of the quadrilateral $DBCE$.

Let N be the centre of gravity of $DBCE$, M that of ADE, O that of ABC.

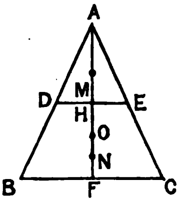

FIG. 93.

$$\therefore \; OM = 3 . ON;$$
$$\text{or, } AO - AM = 3 . ON;$$
$$\text{or, } \frac{2}{3}AF - \frac{1}{3}AF = 3ON;$$
$$\therefore \; ON = \frac{1}{9}AF.$$
$$\therefore \; NF = \frac{1}{3}AF - \frac{1}{9}AF = \frac{2}{9} . AF;$$
$$\therefore \; NF = \frac{4}{9} . HF.$$

40. (1) The given system is equivalent to the system represented in this diagram, if we combine equal parts of the forces, acting at the centres of the outside squares on the right and left, in the centre of each middle square.

2	13	0
0	11	4
2	13	0

FIG. 94.

And this system is equivalent to the system represented in this second diagram, if we combine the forces vertically.

0	0	0
4	37	4
0	0	0

FIG. 95.

∴ the centre of gravity is the middle point of the middle square; that is, the point of intersection of the diagonals.

(2) The new system is equivalent to the system represented in figure 96, if we combine the forces horizontally.

2	4	0
0	11	4
2	13	0

FIG. 96.

And this is equivalent to the system represented in figure 97.

0	0	0
0	27	0
0	9	0

FIG. 97.

Hence, if we take the straight line joining the centres of the squares marked 9 and 1 in the original diagram (since we have forces in the ratio of 3 : 1 acting at the middle point and the lowest point of this line), the centre of gravity of the system will divide the line joining the centres of 9 and 1 in the ratio 5 : 4.

41. Combining horizontally, the system is equivalent to

$$9\tfrac{1}{2} \text{ at } H,\ 6\tfrac{1}{2} \text{ at } G,$$
$$12 \text{ at } K,\ 5 \text{ at } F,$$
$$8\tfrac{1}{2} \text{ at } D,\ 3\tfrac{1}{2} \text{ at } E.$$

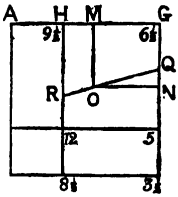

FIG. 98.

Hence if R and Q be the position of the points where these forces, taken vertically, may be collected, a being a side of the square,

$$HR = \frac{8\frac{1}{2} \times a + 12 \times \frac{2a}{3}}{30} = \frac{33a}{60} = \frac{11a}{20},$$

$$GQ = \frac{3\frac{1}{2} \times a + 5 \times \frac{2a}{3}}{15} = \frac{41a}{90}.$$

Also, since O, the centre of gravity of 30 lbs. at R and 15 lbs. at Q, is at the distance from R of $\frac{2}{3}$ RQ,

$$ON = MG = \frac{2}{3}HG = \frac{4}{9}a,$$

$$GN = GQ + \frac{2}{3}(HR - GQ) = \frac{41a}{90} + \frac{2}{3}\left(\frac{11a}{20} - \frac{41a}{90}\right) = \frac{14a}{27}.$$

42. Let a be a side of the large square, N the centre of gravity of the large square, R the centre of gravity of the portion removed, M the centre of gravity of the remainder.

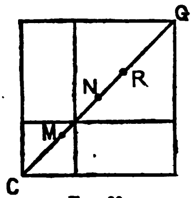

FIG. 99.

Then $MN \times \left(a^2 - \frac{4a^2}{9}\right) = RN \times \frac{4a^2}{9}$;

$$\therefore MN \times 5a^2 = RN \times 4a^2.$$

$$5MN = 4RN$$

$$= 4\,(GN - GR)$$

$$= \frac{2}{3}GC;$$

$\therefore MN = \frac{2}{15}GC$, and $\therefore CM = \frac{11}{30}$ of CG.

43. Let F and E be the middle points of AB, BC; D the centre of gravity of the straight bar; G the centre of gravity of the bent bar.

Then $FB = 4$ feet, and $BE = 3$ feet, and $\therefore FE = 5$ feet.

Then $FG = \frac{3}{7}$ of $FE = \frac{15}{7}$ feet, and $FD = 3$ feet.

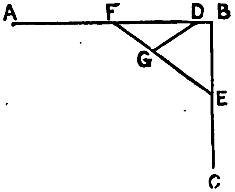

FIG. 100.

Take the formula $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$.

(TRIGONOMETRY, Art. 179.)

Then $\cos DFG = \dfrac{FD^2 + FG^2 - GD^2}{2FD \,.\, FG}$,

$$\text{or, } \frac{4}{5} = \frac{9 + \frac{225}{49} - GD^2}{2 \times 3 \times \frac{15}{7}}$$

$$\text{or, } \frac{4}{5} = \frac{441 + 225 - 49GD^2}{2 \times 3 \times 15 \times 7},$$

whence $GD = \dfrac{9\sqrt{2}}{7}$.

44. O, the centre of gravity of the complete hexagon, is equidistant from the angular points. Let the side omitted be indicated by the dotted line, and let the distance between M the middle point of this line and the opposite side be a.

Then if G be the centre of gravity of the mutilated figure,

$$GO \times 5 = OM \times 1;$$

$$\therefore GO = \frac{OM}{5} = \frac{a}{10}.$$

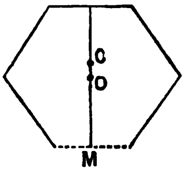

FIG. 101.

45. The centre of gravity of a triangle coincides with the centre of gravity of three equal weights placed at the angular points. Hence the weight sustained by each prop is one-third of the weight of the triangle.

46. Draw a radius CD from any point, and place the weight of 5 lbs. at C. Produce CD to E, so that $ED = \frac{5}{6} \,.\, CD$. Draw the chord AEB at right angles to CE, and place the weights of 3 lbs. each at A and B. These are equivalent to 6 lbs. acting at E, and 6 lbs. at E will balance 5 lbs. at CE, because $DE : CD = 5 : 6$.

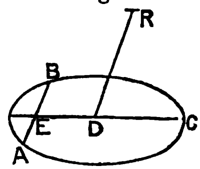

FIG. 102

47. Let the line drawn parallel to the base cut the line AD, joining the vertex of ABC to D, the middle point of BC, into two parts, m and n, of which m is the part nearer to the vertex.

Then area of AEF : area of $ABC = m^2 : (m+n)^2$

EUCLID, VI. XIX.

Let O and G be the centres of gravity of the two triangles, and R the centre of gravity of the quadrilateral $EBCF$.

Then $GR \times \{(m+n)^2 - m^2\} = OG \times m^2$.

$$\therefore \left\{AR - \frac{2(m+n)}{3}\right\}\{2mn + n^2\} = \frac{2}{3}n \times m^2;$$

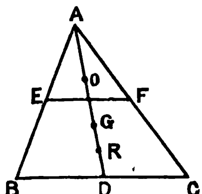

FIG. 103.

$$\therefore AR(2mn + n^2) = \frac{2m^2n}{3} + \frac{2}{3}(2m^2n + 2mn^2 + mn^2 + n^3);$$

$$\therefore AR(2mn + n^2) = \frac{2}{3} \,.\, \{3m^2n + 3mn^2 + n^3\};$$

$$\therefore AR = \frac{2}{3} \cdot \frac{\{(m+n)^3 - m^3\}}{\{(m+n)^2 - m^2\}}.$$

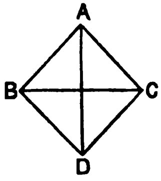

FIG. 104.

48. Let A be the point of suspension. Then, by the conditions of the question, AD bisects BC at right angles. Hence $AB = AC$ (EUCLID, I. IV.); and $\therefore$ the four sides are all equal.

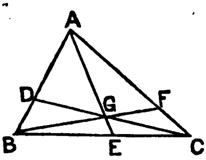

FIG. 105.

49. Place w at A, $2w$ at B, $3w$ at C. w at A and $2w$ at B are equivalent to $3w$ at D, AD being equal to twice DB.

Join DC, and bisect it at G, then G is the centre of gravity required.

$$\text{Also } AG : GE = 5 : 1$$
$$CG : GD = 1 : 1$$
$$BG : GF = 2 : 1.$$

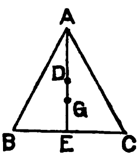

FIG. 106.

50. Let G be the centre of gravity of the lamina, and D the middle point of AE the perpendicular on BC. Let x be the weight placed at A.

$$\text{Then } x \times AD = 6 \times DG;$$
$$\text{or, } x \times \frac{AE}{2} = 6 \times \left(\frac{AE}{2} - \frac{AE}{3}\right);$$
$$\text{or, } \frac{x}{2} = \frac{6}{6}, \text{ and } \therefore x = 2 \text{ lbs.}$$

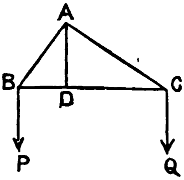

FIG. 107.

51. Let A be the point of suspension. Then AD is perpendicular to BC. Let P and Q be the weights.

$$\text{Then } P : Q = CD : BD.$$
$$\left.\begin{array}{r}\text{Now } BD : BA = BA : BC \\ \text{and } CD : CA = CA : BC\end{array}\right\} \quad \text{EUCLID, VI. XII.}$$
$$\therefore BD = \frac{BA^2}{BC} \text{ and } CD = \frac{CA^2}{BC};$$
$$\therefore P : Q = CA^2 : BA^2.$$

52. At the limit of equilibrium AC is vertical, $\therefore \angle ACD = 45°$.
$\therefore$ the inclination of the plane is 45°.

If the inclination of the plane be greater than 45°, the centre of gravity of the block will be outside the base, and the block will fall.

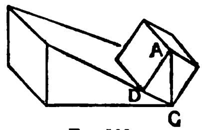

FIG. 108.

53. A is the centre of gravity of the two smaller squares, and D is the centre of gravity of the square on the hypotenuse.

Now the weight of the two squares collected at A is equal to the weight of the larger square collected at D, and therefore the common centre of gravity is at O, which bisects AD, and also bisects the hypotenuse of the triangle.

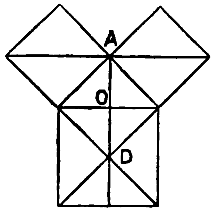

FIG. 109.

EXAMPLES—VII. (p. 87.)

1. Let $AD = x$ feet.

Then $DB = (5 - x)$ feet,

and $4x = 6\ (5 - x)$, $\therefore x = 3$.

Pressure on the fulcrum $= 4$ lbs $+ 6$ lbs $= 10$ lbs.

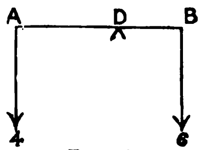

FIG. 110.

2. $DB : AD = 7 : 9$.

$P + Q = 36$ lbs.

Now $P : Q = DB : AD$;

$\therefore P : 36 - P = 7 : 9$, $\therefore 9P = 252 - 7P$;

$\therefore P = 15\frac{3}{4}$ lbs., and $Q = 20\frac{1}{4}$ lbs.

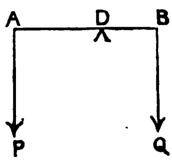

FIG. 111.

3. Let x be the weight of the rod, which may be supposed to act at M, the middle point of the rod.

Then $x : 3 = DN : MD$

$= 4 : 3\frac{1}{2}$;

$\therefore \frac{7}{2}x = 12$, or, $x = \frac{24}{7} = 3\frac{3}{7}$ lbs.

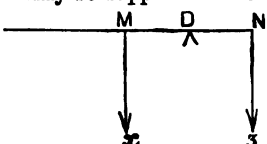

FIG. 112.

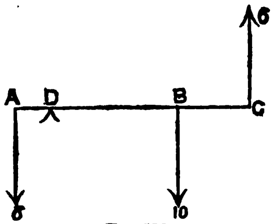

FIG. 113.

4. Let $AD = x$ inches.

Then $6AD + 6CD = 10DB$,

or, $6x + 6\ (10 - x) = 10\ (7 - x)$,

or, $6x + 60 - 6x = 70 - 10x$;

$\therefore\ x = 1$ inch.

Pressure on fulcrum $= (10 + 6 - 6)$ lbs. $= 10$ lbs.

5. Regard B, where the rod rests on the shoulder of one man, as a fulcrum, and let x be the pressure on the shoulder of the other man; w the weight of the rod, acting at its centre of gravity, distant from B one-third of the length of the rod.

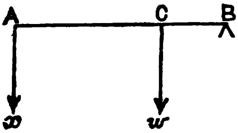

FIG. 114.

Then $x : w = BC : AB$

$= 1 : 3$

$\therefore\ x =$ one-third of w.

Hence pressure on $B =$ two-thirds of w.

6. Let AB be the original length of the lever, D the fulcrum, $DB =$ one-third of AB.

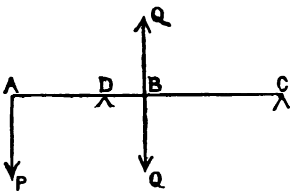

FIG. 115.

Then $Q = 2P$.

Now let Q act vertically upwards at B, and let C be the new position of the fulcrum.

Then $2BC = AC$

$= AB + BC$;

$\therefore\ BC = AB = 3DB$.

7. M is the middle point of the lever.

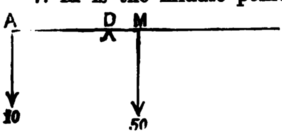

FIG. 116.

Let $AD = x$ feet.

Then $10x = 50\ (3 - x)$,

$10x = 150 - 50x$,

and $\therefore\ x = 2\frac{1}{2}$ feet.

Also pressure on the fulcrum $= 60$ lbs.

8. Collect w, the weight of the beam at the point N round which it balances.

Then $w \times DN = 100 \times DB$,

or, $w \times 8 = 100 \times 2$;

$\therefore\ w = 25$ lbs.

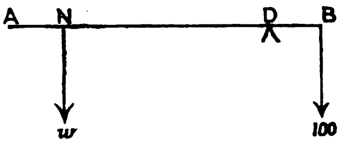

FIG. 117.

9. M is the middle point of the rod, w is the weight of the rod.

Then $2\frac{1}{2} \times AD = w \times MD + 2\frac{2}{3} \times BD$,

or, $2\frac{1}{2} \times 4 = w \times 2 + 2\frac{2}{3} \times 1\frac{1}{2}$,

or, $10 = 2w + 4$, and $\therefore\ w = 3$ lbs.

Also,

pressure on $D = (3 + 2\frac{2}{3} - 2\frac{1}{2})$ lbs. $= 3\frac{1}{6}$ lbs.

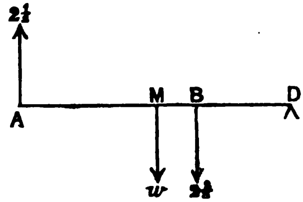

FIG. 118.

10. Regard the peg D as a fulcrum, and let x be the pressure on the peg C.

Then $3 \times AD = x \times CD + 5 \times BD$,

or, $3 \times 5 = 4x + 5 \times 1$;

$\therefore\ x = 2\frac{1}{2}$ lbs.

Hence

pressure on $D = (8 - 2\frac{1}{2})$ lbs. $= 5\frac{1}{2}$ lbs.

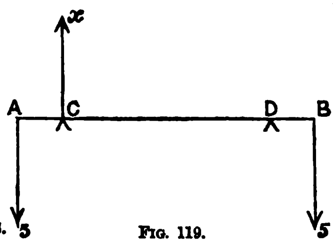

FIG. 119.

Next, let $2p$ be the upward pressure applied at N, a point 2 feet from B.

Then pressure on each peg
$= \frac{1}{2}(8 - 2p) = 4 - p$.

Make D the fulcrum, as before.

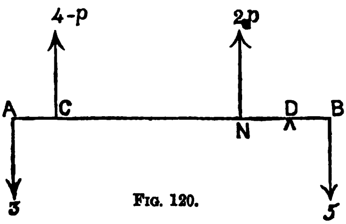

FIG. 120.

Then $3 \times AD = (4 - p) \times DC + 2p \times ND + 5 \times DB$.

$15 = 16 - 4p + 2p + 5$, $\therefore\ 2p = 6$ lbs.

11. Let $DB = x$ inches.

$$\text{Then } 2AD + 7CD + 5ED = 3DB,$$
$$\text{or, } 2(17 - x) + 7(15 - x) + 5(13 - x) = 3x,$$
$$\text{or, } 34 - 2x + 105 - 7x + 65 - 5x = 3x;$$
$$\therefore x = 12.$$

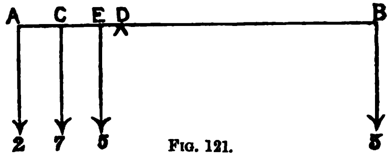

FIG. 121.

12. The advantage will be increased, because the distance of the power from the fulcrum is increased.

13. Let M be the middle point of the bar, and suppose the pressure on the prop B to be twice as great as that on A, the prop that is 1 foot from one end of the bar.

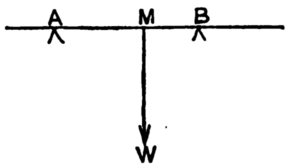

FIG. 122.

Then pressure on $A = \frac{W}{3}$.

Regard B as the fulcrum, and make an upward force of $\frac{W}{3}$ acting at A represent the resistance of A.

$$\text{Then } \frac{W}{3} \times AB = W \times MB$$
$$= W.(AB - AM) = W.(AB - 4);$$
$$\therefore AB = 3AB - 12, \text{ or, } AB = 6 \text{ feet.}$$

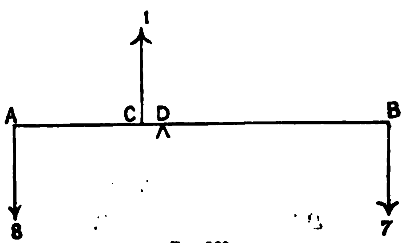

FIG. 123.

14. Let C be the point of application, $CD = x$ inches.

Then

$$8 \times 8 = 7 \times 9 + 1 \times x;$$
$$\therefore x = 1 \text{ inch.}$$

15. $x \times CD = 8 \times AD + 8 \times BD$,

or, $16x = 8\ (18 + 12)$;

$\therefore\ 2x = 30$, or, $x = 15$ lbs

The force, being less than the sum of those which it balances, acts at a mechanical advantage.

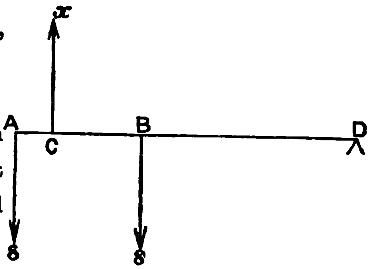

FIG. 124.

16. Make B the fulcrum, and regard the resistance of A as an upward force $= x$ stone. M is the middle point of the beam.

Then $x \times AB = 4 \times DB + 16 \times MB$,

$6x = 4 \times 4 + 16 \times 3$;

$\therefore\ x = 10\frac{2}{3}$ stone;

and $\therefore$ pressure on $B = (20 - 10\frac{2}{3})$ stone $= 9\frac{1}{3}$ stone.

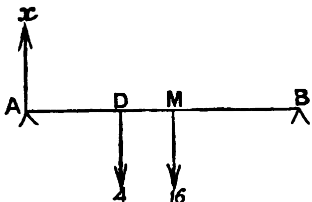

FIG. 125.

17. Suppose the weight supported by the man at B to be p, and the weight supported by the man at A to be rp.

Then the weight of the beam acting at the middle point M is $(r+1)p$.

Regard B as the fulcrum.

Then $rp \times AB = (r+1)p \times MB$.

Let $2x$ be the length of the beam.

Then $rp \cdot (2x - a - b) = (r+1)\ p \cdot (x - b)$,

or, $r\ (2x - a - b) = rx + x - br - b$,

or, $2rx - ra = rx + x - b$,

or $(r-1)x = ra - b$, and $\therefore\ 2x = \dfrac{2(ra - b)}{r - 1}$.

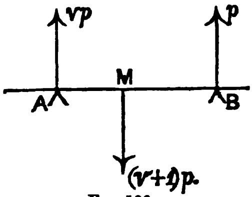

FIG. 126.

18. Let M, N be the middle points of the axes of the cylinders, and O the point where the axes meet.

Now since $MO = \frac{3}{2}$ ft., and $NO = \frac{5}{2}$ ft.,

and since $15 \times \frac{3}{2} = 9 \times \frac{5}{2}$,

it follows the cylinders balance at O, the point of junction.

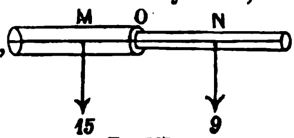

FIG. 127.

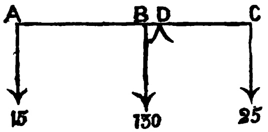

FIG. 128.

19. Let $CD = x$ feet.

Then $15 \times (12 - x) + 130\ (6 - x) = 25x$;

or, $180 - 15x + 780 - 130x = 25x$;

or, $960 = 170x$;

$\therefore\ x = 5\frac{11}{17}$ feet.

20. Let D be the point at which w, a weight equal to the weight of the rod, is placed.

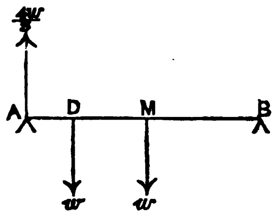

FIG. 129.

Let M be the middle point of the rod.

Then pressure on $A = \frac{2}{3}$ of $(w + w) = \frac{4w}{3}$.

Regard B as the fulcrum, and represent the length of AB by x.

Then $\frac{4w}{3} \times x = w \times (x - AD) + w . \frac{x}{2}$;

$\therefore\ \frac{4x}{3} = x - AD + \frac{x}{2}$;

$\therefore\ AD = \frac{3x}{2} - \frac{4x}{3} = \frac{x}{6} =$ one-sixth of the length of the rod.

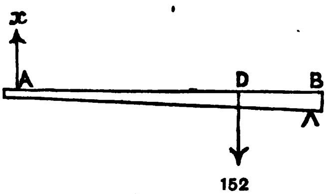

FIG. 130.

21. Regard B as the fulcrum, and let x, equal to the weight sustained by A, act upwards at A.

Then $x \times AB = 152 \times DB$,

or $x \times 38 = 152 \times 14$;

$\therefore\ x = 56$ lbs.;

and $\therefore$ pressure at $B = 96$ lbs.

22. Let w be the weight of the beam, which may be supposed to act at D, round which the beam balanced in its first position.

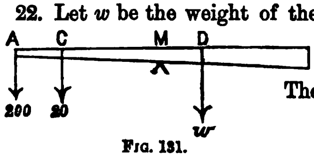

FIG. 131.

Then $200 \times AM + 20 \times CM = w \times MD$;

or, $200 \times 12 + 20 \times 8 = 2w$;

$\therefore\ 2w = 2560$, or, $w = 1280$ lbs.

23. Let M be the middle point, D the fulcrum at a distance of x feet from the end N.

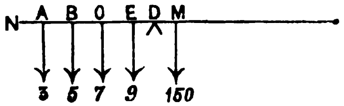

FIG. 132.

Then $3AD+5BD+7CD+9ED=150MD$.

$$\text{Or, } 3(x-2)+5(x-4)+7(x-6)+9(x-8)=150(10-x);$$

$$\therefore 174x=1640; \text{ or, } x=9\tfrac{37}{87} \text{ feet.}$$

24. (1) Let M be the middle point of the lever, W the weight of the lever, D the fixed point, B the end where P, when suspended from it, balances the lever.

Then $W\times MD=P\times DB$.

Let A be the point where P is placed to balance nP acting at B.

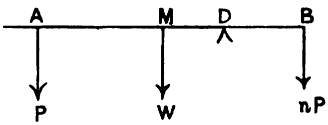

FIG. 133.

Then

$$P\times AD+W\times MD=nP\times DB;$$

$$\text{or, } P\times AD+P\times DB=nP\times DB;$$

$$\text{or, } AD+DB=nDB;$$

$$\therefore AD=(n-1)DB.$$

(2) Here $AD=10\times DB$;

$$\therefore 10=n-1; \text{ or, } n=11.$$

25. Let M be the middle point. It is clear that the additional weight must be attached to some point in AD, and that it may be as small as possible, it must be made to act at A. Let x be the additional weight.

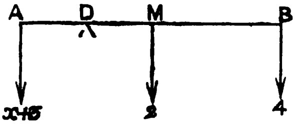

FIG. 134.

Then

$$(x+3)\times AD=2\times DM+4\times DB;$$

$$\text{or, } x+3=2+12, \text{ and } \therefore x=11 \text{ lbs.}$$

26. Let W and $3W$ be the weight of the rods, N and M the middle points of the rods, x the weight attached to W, D the fulcrum, A and B the points in the horizontal line through the fulcrum, through which the vertical lines through N and M pass.

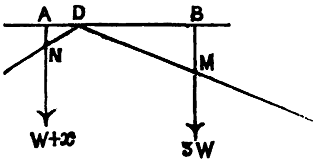

FIG. 135.

Then since $\angle ADN = \angle DBN$, and $\angle DAN = \angle DBM$;

$\therefore$ triangles ADN, BDM are similar.

Then $W+x : 3W = DB : AD$

$= 3 : 1$;

$$\therefore W+x=9W;$$

$$\therefore x=8W.$$

27. Let M be the middle point of the rod, C the first position of the fulcrum, D the second position of the fulcrum, W the weight, and $2x$ the length of the rod.

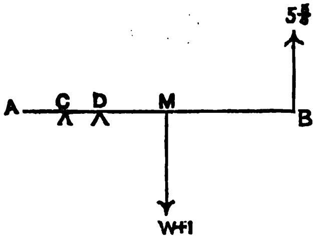

FIG. 136.

$$\text{Then } 5\tfrac{3}{5} \times (2x-2) = W(x-2), \qquad (1)$$

$$\text{and } 5\,(2x-3) = (W+1)(x-3). \qquad (2)$$

$$\text{Hence } \left.\begin{aligned} 56x-56 &= 5Wx-10W, \\ 10x-15 &= Wx-3W+x-3, \end{aligned}\right\} \qquad (3)$$

$$\text{or, } \left.\begin{aligned} 56x-56 &= 5Wx-10W, \\ 50x-75 &= 5Wx-15W+5x-15; \end{aligned}\right\}$$

$$\therefore 6x+19 = 5W-5x+15,$$

$$\text{or, } 5W = 11x+4.$$

Substitute this value of $5W$ in (3), and we get

$$56x-56 = 11x^2-18x-8,$$

and from this we find $x=6$, and $\therefore 2x=12$, and $W=14$.

28. Let W be the weight of the rod, $2x$ the length of the rod, C and D the two positions of the fulcrum.

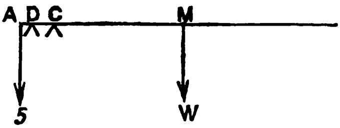

Fig. 137.

Then $\left.\begin{aligned} W(x-2)&=5\times 2 \\ W(x-1)&=11\times 1 \end{aligned}\right\}$

$\left.\begin{aligned} Wx-2W&=10 \\ Wx-W&=11 \end{aligned}\right\}$ and $\therefore$ $W=1$, and $\therefore$ $2x=24$.

29. Let M be the middle point of AB.

Then $12\times AD=8\times DB+x\times DM$,

or, $12\times 11=8\times 4+x\times \frac{7}{2}$;

$\therefore$ $x=28\frac{4}{7}$ lbs.

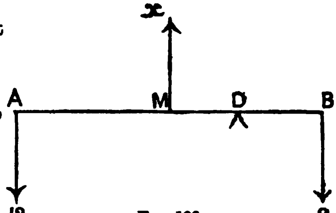

Fig. 138.

30. Let $DB=x$.

Then $9\times 7+7\times 6+5\times 3=3\times 2+5\times 4+7\times 8+x$,

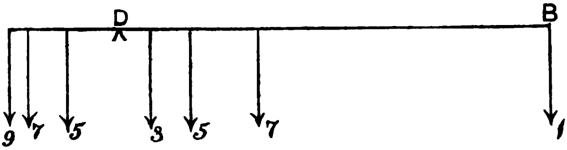

Fig. 139.

or, $63+42+15=6+20+56+x$;

$\therefore$ $x=38$ feet.

31. From D the fulcrum draw DB at right angles to the direction of x, the force acting at the longer arm.

Then since $\angle\, CDB=60°$,

$\therefore$ $CD=2DB$.

Now $x\times DB=6\times AD$,

or, $x\times \frac{5}{2}=6\times 3$, and $\therefore$ $x=7\frac{1}{5}$ lbs.

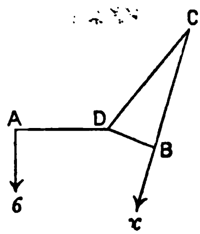

Fig. 140.

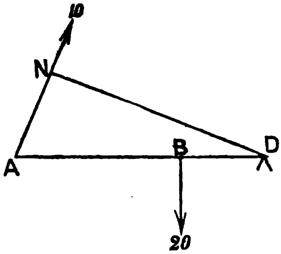

FIG. 141.

32. From D the fulcrum draw DN at right angles to the force of 10 lbs.

Then $AD : DN = 2 : \sqrt{3}$,

and $10 \times DN = 20 \times 4$;

$\therefore DN = 8$, and $\therefore AD = \frac{16}{\sqrt{3}} = \frac{16\sqrt{3}}{3}$;

$\therefore$ the length of lever is $\frac{16\sqrt{3}}{3}$ feet.

33. One way clearly is to reverse the position of the rods, making the 1 lb. weight of the one correspond to the 9 lbs. weight of the other.

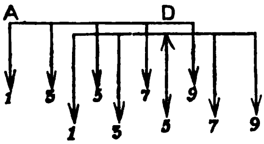

FIG. 142.

Another way is found in the following manner :—

Let D be the fulcrum.

Let x = the distance of the end of the upper rod from the fulcrum.

Then

$$1 \times x + 3(x-1) + 5(x-2) \times 7(x-3) + 1 \times 2 + 3 \times 1$$
$$= 9(4-x) + 7 \times 1 + 9 \times 2,$$

or, $x + 3x - 3 + 5x - 10 + 7x - 21 + 2 + 3 = 36 - 9x + 7 + 18$,

or, $16x - 29 = 61 - 9x$, and $\therefore x = 3\frac{3}{5}$ feet;

$\therefore$ the 1 lb. weight of the upper rod projects $1\frac{3}{5}$ feet beyond the 1 lb. weight of the lower.

34. Draw BD at right angles to the direction of the force of 4 lbs.

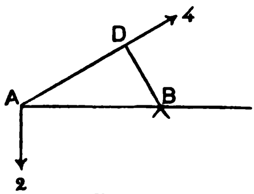

FIG. 143.

Then $2 \times AB = 4 \times BD$

$= 4 \times AB . \sin BAD$;

$\therefore \sin BAD = \frac{1}{2}$, and $\therefore \angle BAD = 30°$.

Also, since the pressure on the fulcrum acts along AB, if R be this pressure,

$4^2 = R^2 + 2^2$, or, $R^2 = 12$,

and $\therefore R = 2\sqrt{3}$ lbs.

35. Let W and $2W$, the weights of the beams, act at M and N their middle points.

Let P be the weight required.

Let $ABDC$ be a horizontal line.

Then $P \times AD + W \times BD = 2W \times DC$.

Now $RD = ND$, and $\therefore AD = DC$.

And $RM = MD$, and $\therefore BD = \frac{1}{2}AD$.

Hence $P \times AD + W \times \frac{1}{2}AD = 2W \times AD$,

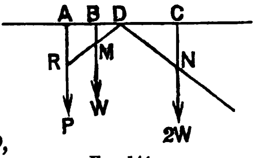

FIG. 144.

$$\text{or, } P + \frac{W}{2} = 2W, \text{ and } \therefore P = \frac{3W}{2}.$$

36. Draw MBN at right angles to the directions of the weights, and BD vertical.

Then since the weights are equal $BM = BN$;

$\therefore AD = DC$. (EUCLID, VI. II. Ex. 1.)

Hence, since ABC is a right angle;

$\therefore DB = AD = DC$. (EUCLID, III. XXXI.)

$\therefore \angle DAB = \angle ABD$, and $\angle DCB = \angle DBC$.

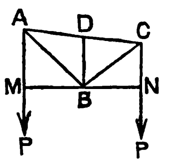

FIG. 145.

37. Let ADB be the lever and D the fulcrum, CDE a line perpendicular to AP, BP, M the middle point of AB, MD a vertical line.

Then since AP, MD, BP are parallel;

$\therefore CD : DE = AM : MB$.

(EUCLID, VI. II. Ex. 1.)

$\therefore CD = DE$, and $\therefore$ there is equilibrium.

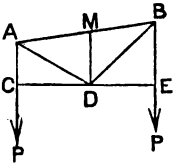

FIG. 146.

38. Let MAN be a horizontal line. Then since AB and AC are equally inclined to the vertical, they are equally inclined to the horizontal;

$\therefore \angle MAB = \angle NAC$;

$\therefore MBA$, NCA are similar triangles.

Then $P : Q = AN : AM$

$= AC : AB$

$= 1 : 2$.

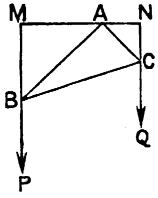

FIG. 147.

39. Let D be the fulcrum, MDN a horizontal line.

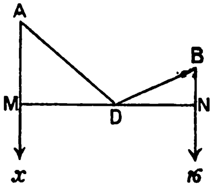

Fig. 148.

Then $DM = \frac{AD}{\sqrt{2}}$,

and $DN = \frac{\sqrt{3}}{2} . DB$;

$\therefore x \times \frac{18}{\sqrt{2}} = 16 \times \frac{\sqrt{3}}{2} \times 12$;

$\therefore x = \frac{16 \times \sqrt{3} \times 12 \times \sqrt{2}}{2 \times 18} = \frac{16\sqrt{6}}{3}$ lbs.

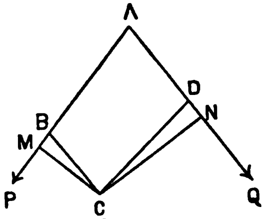

Fig. 149.

40. Let P and Q be the forces acting along AB and AD.

Draw CM, CN perpendicular to AP, BQ.

Then since $\angle ABC = \angle ADC$;

$\therefore \angle CBM = \angle CDN$;

$\therefore$ triangles BMC, DNC are similar;

$\therefore P : Q = CN : CM$
$= CD : BC$.

Examples—VIII. (p. 96.)

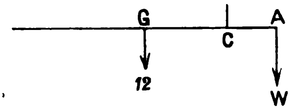

Fig. 150.

1. W will be greatest when the moveable weight is at D, the extremity of the steelyard.

Then $\frac{4}{3}CD \times 12CG = W . CA$,

or, $\frac{4}{3} \times 33 + 12 \times 14 = 5W$,

and $\therefore W = 42\frac{2}{5}$.

2. First, when G the centre of gravity is in the longer arm.
Let P be resting at the graduation marked n.
Then the steelyard says that $W = nP$, or, $W = 2n$ lbs.

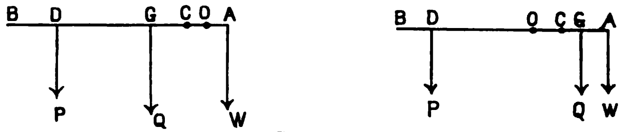

Fig. 151.

But really $2 . CD + Q . CG = W . CA$ (a),

and $Q . CG = 1 . CO$;

$\therefore 2CD + CO = W . CA$, or, $2(OD - CO) + CO = W . CA$.

Also, $OD = nCA$;

$$\therefore 2n \,.\, CA - CO = W \,.\, CA;$$

$$\therefore W = 2n - \frac{CO}{CA}, \text{ and } \therefore W \text{ is less than } 2n.$$

Next, when G lies in CA, the equation (a) becomes

$$2CD = W \,.\, CA + CO,$$

$$\text{or, } 2\,(OD + CO) = W \,.\, CA + CO;$$

$$\therefore 2n \,.\, CA + CO = W \,.\, CA;$$

$$\therefore W = 2n + \frac{CO}{CA}, \text{ and } \therefore W \text{ is greater than } 2n.$$

3. $P \times CD + \frac{P}{p} \times CG = W \times AC.$

Let x = length of rod.

$$P \times \frac{3x}{4} + \frac{P}{p} \times \frac{x}{4} = W \times \frac{x}{4};$$

$$\therefore 3P + \frac{P}{p} = W, \text{ or, } W = \left(\frac{3p+1}{p}\right)P.$$

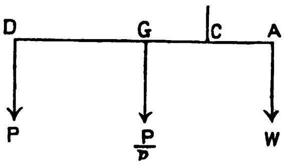

FIG. 152.

4. There is no limit to the *increase* in the magnitude of P, wherever the fulcrum may be, provided that the graduations can be made sufficiently small; but there is a limit to the *decrease* in the magnitude of P, except when the fulcrum is at the centre of gravity, because P may be too small to balance Q when P is made to act at a point in CA.

5. Let R be the weight suspended at Q to make the instrument correct for a moveable weight nP.

Then $P \,.\, CD + Q \,.\, CG = W \,.\, AC$, ... (1)

And $nP \,.\, CD + (Q + R) \,.\, CG = nW \,.\, AC$, ... (2)

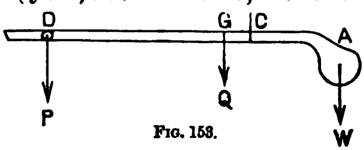

FIG. 153.

Hence, subtracting (1) from (2),

$$(n-1) \,.\, P \,.\, CD + R \,.\, CG = (n-1) \,.\, W \,.\, AC,$$

$$\text{or, } R \,.\, CG = (n-1)\,(W \,.\, AC - P \,.\, CD)$$

$$= (n-1) \,.\, Q \,.\, CG; \quad \text{. . . . . . from (1)}$$

$$\therefore R = (n-1) \,.\, Q.$$

6. If x be the real weight,

$$x : 38 = 20 : 19, \text{ and } \therefore x = 40 \text{ lbs.}$$

7. The arms of the balance are as 14 : 16;

$\therefore$ when the body is placed at the end of the longer arm, apparent weight : 16 = 16 : 14;

$$\therefore \text{apparent weight} = \frac{16 \times 16}{14} = 18\tfrac{2}{7} \text{ ounces.}$$

8. Let x and y be the lengths of the arms, p and q the apparent weights.

Then $px = y$, and $qy = x$;

$$\therefore p + q = \frac{y}{x} + \frac{x}{y},$$

$$\text{or, } \frac{x^2 + y^2}{xy} = \frac{13}{6};$$

$$\therefore x^2 - \frac{13xy}{6} = -y^2,$$

$$\text{or, } x^2 - \frac{13xy}{6} + \frac{169y^2}{144} = \frac{25y^2}{144},$$

$$\text{or, } x - \frac{13y}{12} = \frac{5y}{12};$$

$$\therefore \frac{x}{y} = \tfrac{3}{2} \text{ or } \tfrac{2}{3}.$$

9. He gets 15 ounces for 3s. 9d.

$$\therefore \text{for 16 ounces he pays } \frac{45 \times 16}{15}\text{d., or, 48d.,}$$

that is, he buys at the rate of 4s. per lb.

10. Take the diagram on page 95 of the *Statics*,

$P = 16n$ ounces. Let the graduations run from O to A.

When $W = 1$ ounce, $\dfrac{OC}{AC} = \dfrac{1}{16n}$, and $\therefore OC = \dfrac{1}{16n+1} \cdot AO$.

When $W = 2$ ounces, $\dfrac{OC}{AC} = \dfrac{2}{16n}$, and $\therefore OC = \dfrac{2}{16n+2} AO$, and so on.

EXAMPLES—IX. (p. 99.)

1. Let x be the radius of the axle in inches.

$$\text{Then } 3 : 18 = x : 3 \times 12;$$

$$\therefore x = \frac{3 \times 3 \times 12}{18} = 6 \text{ inches.}$$

2. Let x be the power in lbs.

$$\text{Then } x : 3 = 2 : 6;$$

$$\therefore 6x = 6, \text{ and } \therefore x = 1 \text{ lb.}$$

3. Let x be the power in lbs.

$$\text{Then } x : 12 = 3 : 9;$$

$$\therefore 9x = 36, \text{ and } \therefore x = 4 \text{ lbs.}$$

4. The capstan is a strong cylinder of wood moveable about a vertical axis, with short bars, called handspikes, inserted near the top, by means of which it can be made to revolve. A rope coiled round the capstan is attached to the weight that has to be raised. Hence mechanical advantage is gained, because the distance of the ends of the handspikes from the axis is greater than the distance of the coil of rope from the axis.

Let x be the weight the men can support in cwt.

$$\text{Then } 6 : x = 2 : 5;$$

$$\therefore 2x = 30, \text{ or, } x = 15 \text{ cwt.}$$

5. If a vertical line be drawn through O (see diagram on p. 98 of the *Statics*) and P be made to act at the end of this radius of the wheel, P will produce no pressure on the axle, and therefore the pressure on the axle will in this case be the least possible.

6. Since P and W are equal, it is plain that equilibrium can only exist when the moments of P and W about O are equal, and this will be when the direction of P is a tangent to the axle, as in the diagram.

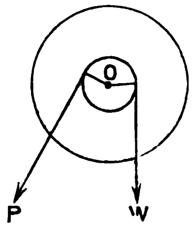

FIG. 154.

7. Let x be the greatest weight that can be supported.

$$\text{Then } 36 : x = 1 : 3;$$
$$\therefore x = 108 \text{ lbs.}$$

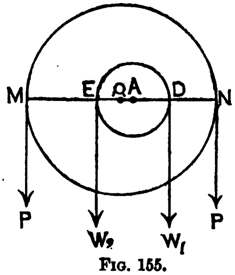

FIG. 155.

8. Let W_1 and W_2 be the two weights.

Let O be the real centre of the wheel, and A the axis of the axle.

$$\text{Then } P \times AM = W_1 \times AD,$$
$$\text{and } P \times AN = W_2 \times EA;$$
$$\therefore P.(AM + AN) = W_1 \times AD + W_2 \times AD;$$
$$\therefore 2P \times OM = (W_1 + W_2)AD.$$

But if O and A coincide,

$$2P \times OM = 2W.AD;$$
$$\therefore W_1 + W_2 = 2W.$$

9. The greatest weight will be supported when W acts at the extremity of a diagonal, and the least when w acts along a side of the square.

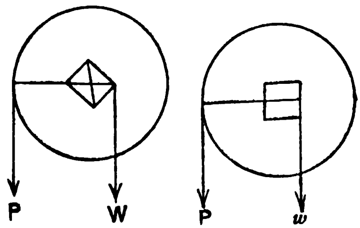

FIG. 156.

$$P : W = \tfrac{1}{2} \text{ diagonal of square : radius of wheel},$$
$$P : w = \tfrac{1}{2} \text{ side of square : radius of wheel};$$
$$\therefore W : w = \text{diagonal of square : side of square}$$
$$= \sqrt{2} : 1.$$

EXAMPLES—X. (p. 105.)

1. Let 2θ be the angle between the strings.

$$\text{Then } 2P.\cos\theta = W,$$
$$\text{and } \therefore 2P.\cos\theta = P, \text{ and } \cos\theta = \tfrac{1}{2}, \text{ or, } \theta = 60^\circ,$$
$$\text{and } \therefore 2\theta = 120^\circ.$$

2. If w be the weight of the pulley,

$$P+P=W+w;$$

$\therefore$ if w be not less than P,

W is not greater than P, and there can be no advantage.

3. $P:W=1:2\cos\theta$

$$=1:2\cos 30^\circ=1:2.\frac{\sqrt{3}}{2}=1:\sqrt{3}.$$

4. $P:8=1:2^3$, and $\therefore P=1$ lb.

5. Let x be the weight of each pulley.

Then $11=\frac{32}{8}+\frac{x}{2}+\frac{x}{4}+\frac{x}{8}$;

or, $88=32+7x$, and $\therefore x=8$ lbs.

6. $P=\frac{5}{8}+\frac{3}{2}+\frac{3}{4}+\frac{3}{8}$;

$\therefore 8P=5+12+6+3$, and $\therefore P=3\frac{1}{4}$ lbs.

7. $P=\frac{160}{16}+\frac{1}{2}+\frac{2}{4}+\frac{4}{8}+\frac{8}{16}$;

$\therefore P=10+2$, or, $P=12$ lbs.

8. $P=\frac{2}{8}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$;

$\therefore P=\frac{16}{8}=2$, or, $P=2$ lbs.

9. $P=\frac{W}{2}+\frac{W}{4}+\frac{W}{8}$;

$\therefore 8P=7W$, or, $P:W=7:8$.

10. Let x be the weight of each pulley in lbs.

Then $7\frac{7}{8}=\frac{56}{8}+\frac{x}{2}+\frac{x}{4}+\frac{x}{8}$;

or, $63=56+7x$, and $\therefore x=1$ lb.

11. Let x be the weight of each pulley.

Then $W=\frac{W}{8}+\frac{x}{2}+\frac{x}{4}+\frac{x}{8}$;

or, $8W=W+7x$, and $\therefore x=W$.

12. Let the number of pulleys be n.

$$\text{Then } W=w+p,$$

$$\text{and } p=\frac{w+p}{2^n}, \text{ or, } 2^n \cdot p-p=w;$$

$\therefore w=(2^n-1)p$, and 2^n-1 is necessarily an *odd* number, because all powers of 2 are *even* numbers.

13. $2=\frac{W}{16}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$;

$$\text{or, } 32=W+8+4+2+1, \text{ and } \therefore W=17 \text{ lbs.}$$

14. Pressure on beam $=P'+2P'+4P'+\ \dots\ 2^{n-1}.P'$

$$=(2^n-1)P';$$

$$\therefore W'=(2^n-1)P';$$

$$\text{also, } W=2^n.P;$$

$$\therefore \frac{W}{P}-\frac{W'}{P'}=2^n-(2^n-1)=1.$$

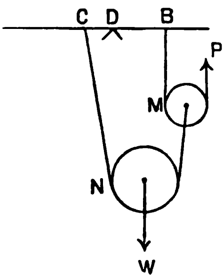

FIG. 157.

15. Let D be the fixed point.

The beam is acted on by two parallel forces,

one in $BM=P$,

one in $CN=2P$;

$\therefore$ there is equilibrium when

$$BD:CD=2P:P$$

$$=2:1.$$

16. $P:8+112=1:6$;

$$\therefore 6P=120, \text{ and } \therefore P=20 \text{ lbs.}$$

17. $2:W+8=1:7$;

$$\therefore 14=W+8, \text{ or, } W=6 \text{ lbs.}$$

18. Let W be the weight of the man, P the power he exerts.

$$\text{Then } P:\frac{W}{2}=1:7;$$

$$\therefore P=\frac{W}{14}.$$

$$\text{Hence pressure on floor}=W-\frac{W}{14}=\frac{13W}{14}.$$

19. $P : W+3P=1:6$,
$$\text{or, } 6P=W+3P, \text{ and } \therefore W=3P.$$

20. Let w be the weight of the lower block.
$$\text{Then } 3:10+w=1:4,$$
$$\text{or, } 12=10+w, \text{ and } \therefore w=2 \text{ lbs.}$$

21. Let n be the number of pulleys at the lower block.

If the string be fastened at the upper block, the number of strings at the lower block is $2n$.

If the string be fastened at the lower block, the number of strings at the lower block is $2n+1$.

In the first case, $p:w+p=1:2n$, and $\therefore w=(2n-1)p$.

In the second case, $p:w+p=1:2n+1$, and $\therefore w=2np$.

22. Let D be the point of suspension.

Then, referring to the diagram on page 104 of the *Statics*, the action of the strings on the bar will be represented by three equidistant vertical forces, P, $2P$, $4P$.

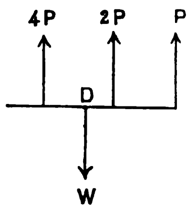

FIG. 158.

Let $2a$ be the length of the bar, x the distance of D from the force $4P$.

When there is equilibrium, the bar being horizontal, taking moments round D,
$$4P\times x=2P\times(a-x)+P\times(2a-x),$$
$$\text{or, } 4x=2a-2x+2a-x;$$
$$\therefore 7x=4a, \text{ or, } x=\tfrac{4}{7}a.$$

23. Taking the diagram on page 104 of the *Statics*, extended to six pulleys, but omitting the string RA, and taking w for the weight of each pulley,

tension of first string, $SB=w$,
tension of second string, $TC=3w$,
tension of third string $=7w$,
tension of fourth string $=15w$,
tension of fifth string $=31w$;
$$\therefore w+3w+7w+15w+31w=W;$$
$$\therefore w:W=1:57.$$

EXAMPLES—XI. (p. 112.)

1. Since $5^2+12^2=169=13^2$.

$$P:W=\text{height of plane}:\text{length of plane}$$
$$=5:13;$$
$$\therefore P=\frac{3\times 5}{13}\text{ tons}=1\tfrac{2}{13}\text{ tons.}$$

2. Since $3^2+4^2=25=5^2$,

$$W:R=\text{length of plane}:\text{base of plane}$$
$$=5:4;$$
$$\therefore R=\frac{10\times 4}{5}\text{ lbs.}=8\text{ lbs.}$$

3. W will be represented by 5;

$$\therefore R=\tfrac{3}{5}W,\text{ and }P=\tfrac{4}{5}W.$$

4. The height of plane will be represented by 3.

$$P:W=\text{height of plane}:\text{base of plane},$$
$$P:12=3:4;$$
$$\therefore P=\tfrac{36}{4}\text{ lbs.}=9\text{ lbs.}$$

5. $P:R=$height of plane : length of plane;

$$\therefore 1:2=\text{height}:\text{length};$$
$$\therefore 1:2:\sqrt{3}=\text{height}:\text{length}:\text{base}.$$

Then $P:W=$height : base

$$=1:\sqrt{3};$$
$$\therefore W=\sqrt{3}\text{ lbs.};$$

and if θ be the inclination of the plane,

$$\sin\theta=\tfrac{1}{2},\text{ or, }\theta=30^\circ.$$

6. Let P and P' be the two forces,

$$\text{height}:\text{base}=P:W$$
$$=15:20=3:4;$$
$$\therefore \text{height}:\text{base}:\text{length}=3:4:5.$$

Hence $P':20=3:5$;

$$\therefore P'=12\text{ lbs.}$$

7. The resistance to motion arising from the rails is (50×10) lbs., and therefore the tension of the rope is thus relieved to the extent of 500 lbs.

Also $P : 50 = 1 : 30$;

$$\therefore P = \tfrac{50}{30} \text{ tons};$$

$$\therefore \text{strain on rope} = (\tfrac{50}{30} - \tfrac{500}{2240}) \text{ tons} = 1\tfrac{229}{336} \text{ tons}.$$

8. $P : W =$ height of plane : base of plane,

and $\therefore$ if a be the inclination of the plane,

$$\tan a = \frac{P}{W} = \frac{15}{5\sqrt{3}} = \sqrt{3}, \text{ and } \therefore a = 60°.$$

9. $P : W =$ height of plane : length of plane;

$$\therefore P : 2\sqrt{2} = 1 : \sqrt{2};$$

$$\therefore P = 2 \text{ lbs.}$$

10. $P : W =$ height of plane : base of plane;

$$\therefore P : 56 = 1 : 1;$$

$$\therefore P = 56 \text{ lbs.}$$

Hence any force greater than 56 lbs. will move the weight.

11. $P : W =$ height of plane : base of plane;

$$\therefore P : 12 = \sqrt{3} : 1;$$

$$\therefore P = 12\sqrt{3} \text{ lbs.}$$

12. $P : W =$ height of plane : base of plane,

and the angle of inclination is 30°;

$$\therefore P : 10 = 1 : \sqrt{3};$$

$$\therefore P = \frac{10}{\sqrt{3}} \text{ lbs.} = \frac{10\sqrt{3}}{3} \text{ lbs.}$$

13. Let P and $2P$ be the forces.

$$\text{Then } \frac{P}{W} = \frac{\text{height of plane}}{\text{length of plane}},$$

$$\text{and } \frac{2P}{W} = \frac{\text{height of plane}}{\text{base of plane}};$$

$$\therefore \tfrac{1}{2} = \frac{\text{base of plane}}{\text{length of plane}};$$

$\therefore$ angle of inclination is 60°.

14. Let P be the tension of the string AW.

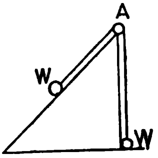

FIG. 159.

Then $P : W = \text{height} : \text{length}$
$= 1 : \sqrt{2}$;

$$\therefore P = \frac{W}{\sqrt{2}}.$$

Hence pressure on the ground

$$= W - \frac{W}{\sqrt{2}} = W\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) = W\left(\frac{2-\sqrt{2}}{\sqrt{2}}\right)$$

15. $P : W = \text{height} : \text{length}$;

$$\therefore P : 4 = 1 : 2, \text{ or, } P = 2 \text{ lbs.}$$

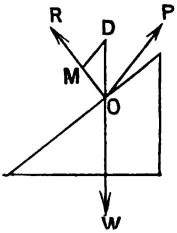

FIG. 160.

16. Let $\angle POD = \angle ROD$.

Draw DM parallel to OP.

Then the sides of MDO are parallel to P, W, R.

Also $\angle MDO = \angle DOP$
$= \angle DOM$.

Now $P : R = DM : OM$.

But $DM = OM$, and $\therefore P = R$.

17. Let P_1 be the power acting parallel to the plane, P_2 the power acting horizontally.

Then $P_1 : W = \text{height} : \text{length}$,
and $P_2 : W = \text{height} : \text{base}$;
$\therefore P_1 \times \text{length} = P_2 \times \text{base}$.
Now, the length is greater than the base;
$\therefore P_1$ is less than P_2.

18. The string will just support the weight when
height of plane : length of plane $= 10 : 20$
$= 1 : 2$,
and in this case the angle of inclination is 30°.
$\therefore$ as soon as the inclination is greater than 30°, the height of the plane is increased, a power greater than 10 lbs. is required, and the string breaks.

19. The tensions of the parts of the string AW and AW' are the same; represent each by P.

Then $\dfrac{W}{P}=\dfrac{AB}{AD}$,

and $\dfrac{W'}{P}=\dfrac{AC}{AD}$;

$\therefore \dfrac{W}{W'}=\dfrac{AB}{AC}$.

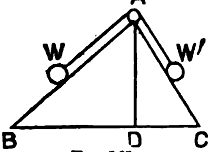

FIG. 161.

20. The power acts first along the plane and then horizontally, and taking $2a$ and a as the angles of inclination, and $2W$ and $3W$ as the weights supported in each case.

$$\frac{P}{2W}=\sin 2a$$

$$\frac{P}{3W}=\tan a;$$

$$\therefore 2\sin 2a=3\tan a;$$

$$\therefore 4\sin a.\cos a=\frac{3\sin a}{\cos a};$$

$$\therefore \cos^2 a=\tfrac{3}{4}, \text{ or, } \cos a=\frac{\sqrt{3}}{2};$$

$$\therefore a=30^\circ, \text{ and } \therefore 2a=60^\circ.$$

21. Let a be the inclination of the plane.

$$\text{Then } \frac{10}{W}=\frac{\text{height}}{\text{length}},$$

$$\text{and } \frac{20}{W}=\frac{\text{height}}{\text{base}};$$

$$\therefore \frac{\text{base}}{\text{length}}=\tfrac{10}{20}=\tfrac{1}{2};$$

$\therefore$ a is an angle of 60°.

22. Height of plane : length of plane = 12 : 20 = 3 : 5;

$\therefore$ height : length : base = 3 : 5 : 4.

Now, horizontal force : 20 = 3 : 4;

$\therefore$ horizontal force = 15 lbs.,

and 15 : 12 = 5 : 4.

Again—

Pressure in first case : $20 = 4 : 5$;

$\therefore$ pressure in first case $= 16$ lbs.

Pressure in second case : $20 = 5 : 4$;

$\therefore$ pressure in second case $= 25$ lbs.,

and $25 : 16 = 5^2 : 4^2$.

23. Height : length : base $= 1 : 2 : \sqrt{3}$,

and hence the angle of inclination is 30°.

24. $BC : AC = 3 : 5$;

$\therefore BC : AC : AB = 3 : 5 : 4$.

Hence if AB become the height of the plane,

3 lbs. : weight supported $= 4 : 5$;

$\therefore$ weight supported $= 3\frac{3}{4}$ lbs.

25. The force required to keep W at rest, acting along the plane, is $W \cdot \sin\alpha$.

Hence the force acting down the plane must be $W \cdot \tan\alpha - W \cdot \sin\alpha$, for then the resultant force tending to pull the body upwards along the plane will be

$$W \tan\alpha - (W \cdot \tan\alpha - W \cdot \sin\alpha) = W \cdot \sin\alpha.$$

26. There will be equilibrium when P and W make equal angles with the direction of R, each angle being evidently α, the angle of inclination of the plane.

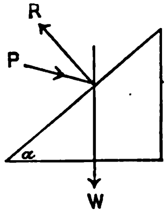

FIG. 162.

Hence $\frac{R_1}{\sin 2\alpha} = \frac{W}{\sin\alpha}$ (Art. XXXIX. of *Statics*.)

and $\therefore R_1 = 2W \cdot \cos\alpha$.

But when P acts parallel to the plane,

$$R = W \cdot \cos\alpha.$$

Hence $R_1 = 2R$.

MISCELLANEOUS EXERCISES (p. 121).

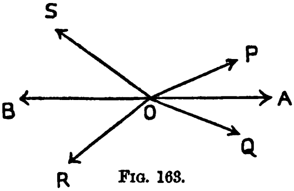

FIG. 163.

1. OA, the direction of the resultant of the equal forces P and Q, bisects the angle POQ.

OA is opposite the resultant of the equal forces R and S, and if OB be the direction of this resultant, it bisects the angle ROS.

$$\therefore \angle^s AOQ, QOR, ROB = 2 \text{ rt. } \angle^s = \angle^s AOP, POS, SOB;$$
$$\therefore \angle QOR = \angle POS.$$

The four forces are not necessarily equal, for the magnitude of the resultants of each pair, depending partly on the magnitude of the forces, and partly on the angle between the directions of the forces, may be equal, without equality existing between the four forces.

2. $P^2 + Q^2 + 2PQ \cdot \cos\theta = R^2 = P^2 + Q^2 + 2PQ \cdot \cos(45^\circ - \theta);$
$$\therefore \cos\theta = \cos(45^\circ - \theta);$$
$$\therefore \cos\theta = \cos 45^\circ \cdot \cos\theta + \sin 45^\circ \cdot \sin\theta;$$
$$\therefore \cos\theta = \frac{1}{\sqrt{2}} \cdot \cos\theta + \frac{1}{\sqrt{2}} \cdot \sin\theta;$$
$$\therefore \sqrt{2} \cdot \cos\theta = \cos\theta + \sin\theta;$$
$$\therefore \sqrt{2} = 1 + \tan\theta, \text{ or, } \tan\theta = \sqrt{2} - 1.$$

3. From A in RO produced draw AB parallel to P, meeting OQ in B.

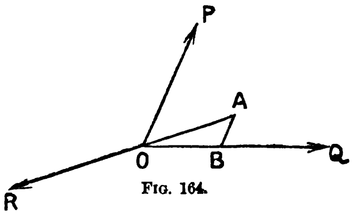

FIG. 164.

Then if in magnitude,

R, Q, P are in descending order;
AO, OB, BA are in descending order;
$\angle OBA$, $\angle BAO$, $\angle AOB$ are in descending order;
$\therefore \angle POQ$, $\angle POR$, $\angle ROQ$ are in ascending order.

4. ML, NO, AC are equivalent to MP, MB, ND, NP, AC,
which are equivalent to CN, ND, CM, MB, AC,
which are equivalent to CD, CB, AC,
which are equivalent to CA, AC,
which represent forces in equilibrium.

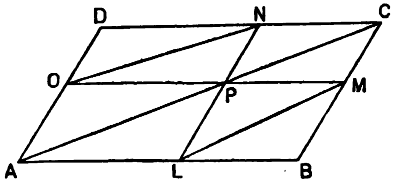

FIG. 165.

5. Let h be the height of the steeper plane, b the base, a the length, P_1 and P_2 the powers employed.

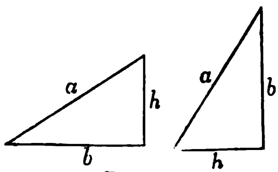

FIG. 166.

Now since the power on the steeper plane is equal to the resistance on the other, the height and base of the second are respectively equal to the base and height of the first.

Then since three times resistance on steeper = resistance on the other.

$$\frac{3\sqrt{a^2-h^2}}{a}=\frac{h}{a};$$

$$\therefore 9(a^2-h^2)=h^2, \text{ or, } 3a=\sqrt{10}\,.\,h.$$

Then $P_1 : W = h : a = 3 : \sqrt{10}$;

$P_2 : W = \sqrt{a^2-h^2} : a = h : 3a = 1 : \sqrt{10}$.

6. Let O be the given point within or without the triangle, G the centre of gravity.

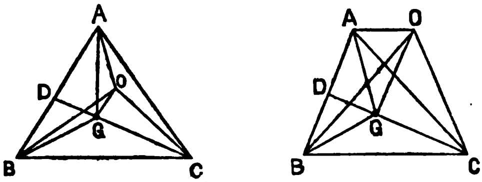

FIG. 167.

Then GO is the resultant of GB, BO,

GO is the resultant of GC, CO,

GO is the resultant of GA, AO;

$\therefore$ three times GO is equivalent to GA, GB, GC, AO, BO, CO.

Now GA, GB are exactly counteracted by GC, for if we produce CG to D, the middle point of AB,

$$GA,\ GB \text{ have a resultant} = 2GD = CG;$$

$$\therefore \text{three times } GO \text{ is equivalent to } AO,\ BO,\ CO.$$

7. Let R and O be the centres of gravity of AMN and $MBCN$, G the centre of gravity of ABC.

Let $AO = x$, $AD = h$.

Now area of $MBCN = 3$ times area of AMN;

$$\therefore OG \times 3 = RG \times 1.$$

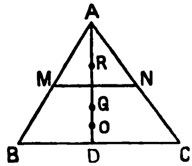

Fig. 168.

Now $OG = x - \dfrac{2h}{3}$

$$RG = AG - AR = \tfrac{2}{3}h - \tfrac{2}{3}\cdot\frac{h}{2} = \frac{h}{3};$$

$$\therefore 3x - 2h = \frac{h}{3}, \text{ and } \therefore x = \frac{7}{9}h.$$

8. Let $AC = 3$ feet, $CB = 4$ feet, $AB = 5$ feet.

Then $\angle ACB$ is a right angle.

Draw DN parallel to AC.

Let t_1, t_2 be the tensions of CB, CA.

Then $CN : ND : DC = t_1 : t_2 : 25$.

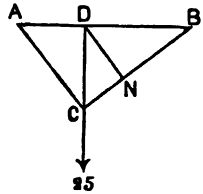

Fig. 169.

But, by similar triangles,

$$CN : ND : DC = AC : CB : BA$$

$$= 3 : 4 : 5;$$

$$\therefore t_1 : t_2 : 25 = 3 : 4 : 5;$$

$$\therefore t_1 = 15 \text{ lbs., and } t_2 = 20 \text{ lbs.}$$

9. The direction of W, the weight of the rod BC, bisects the angle BAC.

The tensions of AB, AC are equal, let each be represented by t.

Then $W^2 = t^2 + t^2 + 2t \cdot t \cdot \cos BAC$

$= t^2 + t^2 + 2t^2 \cdot \cos 60°$

$= t^2 + t^2 + 2t^2 \cdot \frac{1}{2}$

$= 3t^2$;

$\therefore W = \sqrt{3} \cdot t$;

$\therefore$ tension of each string $= \frac{W}{\sqrt{3}} = \frac{W \cdot \sqrt{3}}{3}$.

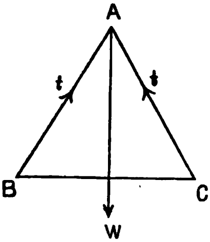

FIG. 170.

10. Let O be the point on which the forces P, Q, R acting are at rest.

$$\text{Then } POR = (180° - 30°) = 150°.$$

$$\text{And } R : Q = \sin POQ : \sin POR; \qquad \text{(Art. XXXIX.)}$$

$$\therefore \sqrt{3} : 1 = \sin POQ : \tfrac{1}{2};$$

$$\therefore \sin POQ = \frac{\sqrt{3}}{2};$$

$$\therefore POQ = 60°, \text{ or } 120°.$$

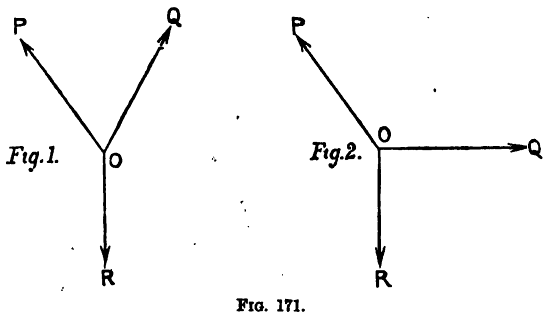

FIG. 171.

Hence $QOR = (360° - 150° - 60°) = 150°$, . . . FIG. 1.

or $QOR = (360° - 150° - 120°) = 90°$; . . . FIG. 2.

$\therefore P : Q = \sin 150° : \sin 150°$, and $\therefore P = Q$,

or $P : Q = \sin 90° : \sin 150°$, and $\therefore P = 2Q$.

11. First, when P acts horizontally

$$\frac{P}{W}=\frac{BC}{AC}=\tan 30^\circ=\frac{1}{\sqrt{3}}.$$

Next, when the direction of P makes an angle of 30° with the plane, resolving in the direction of the plane, there is equilibrium when

$$P\,.\cos 30^\circ=W\,.\cos 60^\circ,$$

$$\text{or }\frac{P}{W}=\frac{\cos 60^\circ}{\cos 30^\circ}=\frac{1}{2}\div\frac{\sqrt{3}}{2}=\frac{1}{\sqrt{3}},$$

which is the relation between P and W established in the first case.

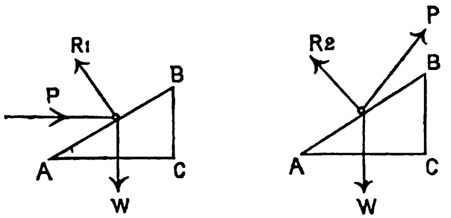

FIG. 172.

Again, if R_1 and R_2 be the pressures on the plane,

$$\frac{R_1}{P}=\frac{\sin 90^\circ}{\sin 150^\circ}=\frac{\sin 90^\circ}{\sin 30^\circ}=2$$

$$\frac{R_2}{P}=\frac{\sin 150^\circ}{\sin 150^\circ}=1\,;$$

$$\therefore R_2=\frac{R_1}{2}.$$

12. $P:Q:R=\sin 120^\circ:\sin 105^\circ:\sin 135^\circ$ (Art. XXXIX.)

$$=\sin 60^\circ:\sin 75^\circ:\sin 45^\circ$$

$$=\frac{\sqrt{3}}{2}:\frac{\sqrt{3}+1}{2\sqrt{2}}:\frac{1}{\sqrt{2}}$$

$$=\frac{\sqrt{6}}{2\sqrt{2}}:\frac{\sqrt{3}+1}{2\sqrt{2}}:\frac{2}{2\sqrt{2}}$$

$$=\sqrt{6}:\sqrt{3}+1:2.$$

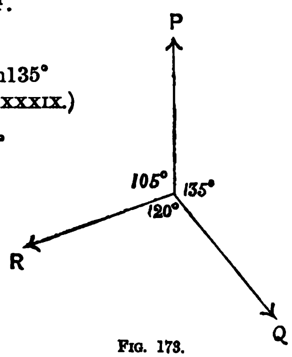

FIG. 173.

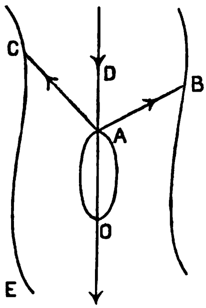

FIG. 174.

13. Let AO be the boat, AB the mooring chain, C the stake by which the rope AC is fastened. Let the current act in the direction DA.

Now, suppose AB to be cut. Then the boat is acted on by two forces, one acting along AO, the other along AC, and the resultant of these forces acts towards the bank CE, to which the boat will therefore swing.

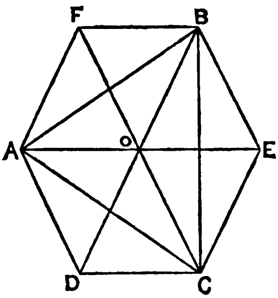

FIG. 175.

14. Let oA, oB, oC represent the forces.

Join AB, BC, CA.

Complete the parallelograms $AoBF$, $BoEC$, $AoCD$.

Then $\square AoBF = \square FoBE$

(EUCLID, I. XXXVI.)

$= \square oBEC$;

$\therefore \triangle AoB = \triangle BoC$.

Similarly, each of these triangles $= \triangle AoC$.

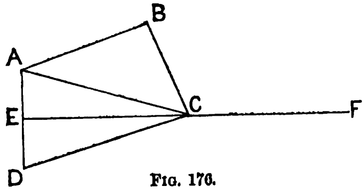

FIG. 176.

15. AB, BC have a resultant AC.

$\therefore$ AB, BC, DC are equivalent to AC, DC, of which the resultant is $2EC$, *i.e.* ECF.

16. Since the figure is symmetrical, pressure on B is equal to pressure on C.

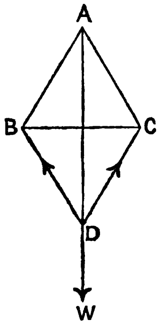

FIG. 177.

Let P be the pressure on each of these pegs.

Then the tensions of BD, CD will each equal P, and $\angle BDC = 60°$.

$$\therefore W^2 = P^2 + P^2 + 2P \cdot P \cdot \cos 60° = 3P^2;$$

$$\therefore P = \frac{W}{\sqrt{3}}, \text{ or, } P = \frac{W \cdot \sqrt{3}}{3}.$$

Also, pressure on A is clearly equal to W.

17. The three forces acting on the sphere are the tension of the string AO, the pressure of the wall acting along the radius BO, at right angles to AB, and the weight of the sphere acting vertically through O, and the lines of action of these forces are parallel to the sides of the triangle AOB.

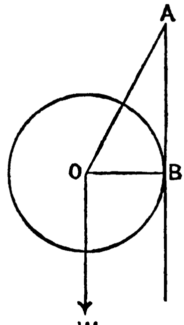

FIG. 178.

$$\therefore R : T : W = OB : OA : AB$$
$$= r : l : (l^2 - r^2)^{\frac{1}{2}}.$$

18. Let T be the tension of the string, a the inclination of each plane, R the pressure on each plane, acting as a normal to the sphere.

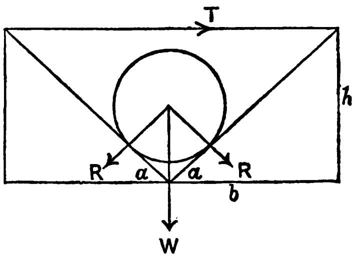

FIG. 179.

Resolving vertically, for the equilibrium of the sphere,

$$W = 2R \cdot \cos a.$$

Resolving horizontally, for the equilibrium of either plane,

$$T = R \cdot \sin a;$$

$$\therefore T = \tan a \cdot \frac{W}{2} = \frac{h}{b} \cdot \frac{W}{2}.$$

19. Let AC be the post, B its middle point,

$\angle DBC = 30°$, and $\therefore \angle BDC = 60°$.

The angle between the two sets of telegraph wires is 120°, and if R be the resultant of the equal tensions of the wires, each $= T$,

$$R^2 = T^2 + T^2 + 2T^2 \cdot \cos 120° ;$$

$$\therefore R^2 = T^2, \text{ and } \therefore R = T.$$

Let t be the tension of rope BD.

Then, taking moments round C,

$$R \times AC = t \times BC \cdot \cos 60° ;$$

$$\therefore T \times AC = t \times \frac{AC}{2} \times \tfrac{1}{2}, \text{ and } \therefore t = 4T.$$

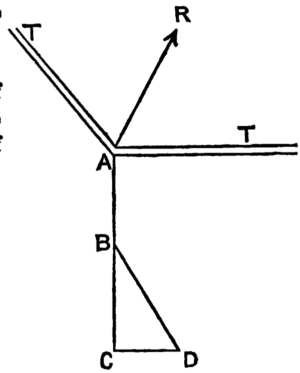

Fig. 180.

20. Let $ABCD$ be the quadrilateral, and $BF = CE$, $DG = AE$.

Bisect BC in O. Join OA, OG, OD.

Let H and K be the centres of gravity of ABC and DBC. Join HK, cutting OG in L.

Then $\because OH = \tfrac{1}{3}OA$, and $OK = \tfrac{1}{3}OD$;

$\therefore HK$ is parallel to AD ;

$\therefore OL = \tfrac{1}{3}OG$, and L is the centre of gravity of triangle EFG. Draw AP and DQ perpendicular to CB.

Then triangle ABC : triangle $DCB = AP : DQ$

$= AE : DE$

$= DG : AG$

$= KL : LH$.

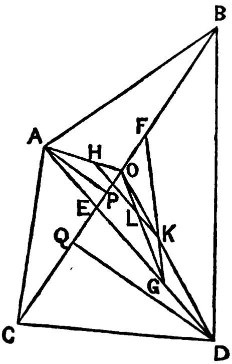

Fig. 181.

Therefore the triangles ABC and DCB will balance about L, or the centre of gravity of the quadrilateral is L ; and this is also the centre of gravity of the triangle EFG.

21. Let A, B, C be the three pegs.

$$\angle BAC = 120° ;$$

$$\therefore \angle ABC = \angle ACB = 30°.$$

Then pressure on $A = 2W \cdot \cos 60° = W$,

pressure on $B = 2W \cdot \cos 30° = W \cdot \sqrt{3}$

$=$ pressure on C.

The vertical pressure on $A = W$ acting upwards.
The vertical pressure on both B and C

$$= W + W.\cos 60° = \frac{3W}{2} \text{ acting downwards.}$$

The reactions which support the weights will be equal and opposite, so that we shall have, in order to support the weights, $\frac{3W}{2}$ at both B and C acting upwards, and W acting downwards at A;
or 3 W upwards, and W downwards, or $2W$ acting upwards, which will just support the weights.

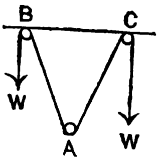

FIG. 182.

22. Resolving along Ox and Oy, of which Ox coincides with BOE, observing that $\cos 60° = \frac{1}{2}$ and $\sin 60° = \frac{\sqrt{3}}{2}$

$$X = 5 + 6.\tfrac{1}{2} - 1.\tfrac{1}{2} - 2 - 3.\tfrac{1}{2} + 2$$
$$= 5 + 3 - \tfrac{1}{2} - 2 - \tfrac{3}{2} + 2 = 6$$
$$Y = 6.\frac{\sqrt{3}}{2} + 1.\frac{\sqrt{3}}{2} - 3.\frac{\sqrt{3}}{2} - 4.\frac{\sqrt{3}}{2}. = 0.$$

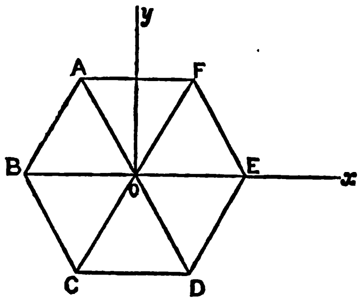

FIG. 183.

Hence the resultant acts along OE, and its magnitude is 6 lbs.

23. Let M be the middle point of the rod, W the weight of the rod, T the tension of each of the strings AD, BE.

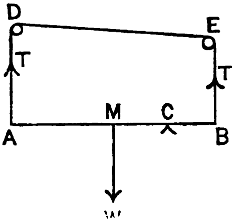

FIG. 184.

Take moments about C, and

$$T.AC=T.BC+W.MC;$$

$$\text{or, } T.(AM+MC)=T.(BM-MC)+W.MC;$$

and $\therefore$ since $T.AM=T.BM$

$$T.MC=W.MC-T.MC;$$

$$\therefore T=W-T,$$

$$\text{or } T=\frac{W}{2}.$$

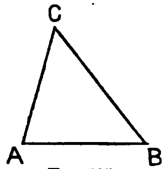

FIG. 185.

24. Let ABC be any one of the triangles, and AC, CB represent forces tending from A, and AB a force tending towards B, it is plain that the resultant of AC, CB will be represented by AB, and $\therefore$ the resultant of AC, CB, AB will be represented by $2AB$.

25. Since the angles AEB, ADB are right angles, a circle described on AB as diameter passes through E and D;

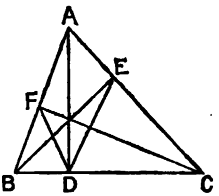

FIG. 186.

$$\therefore \angle ABE=\angle ADE,$$

and since the angles AFC, ADC are right angles, a circle described on AC as diameter passes through F and D;

$$\therefore \angle ACF=\angle ADF.$$

Now $\angle ABE=\angle ACF$,

by similar $\triangle$s ABE, ACF;

$\therefore \angle ADE=\angle ADF$, and $\therefore AD$ bisects $\angle FDE$,

$\therefore$ forces are equal.

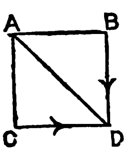

FIG. 187.

26. Let AB, AC be the arms of the lever.

Let the forces P, P act along BD.

Then the pressure on A will be represented by AD.

$$\text{Now } AD^2=2BD^2;$$

$$\therefore AD=\sqrt{2}.BD,$$

$$\text{or pressure on } A=\sqrt{2}.P.$$

ELEMENTARY HYDROSTATICS.

KEY.

EXAMPLES—I. (p. 8.)

(1.) $1\frac{1}{8} : 64 = 1$ ton : weight supported ;

$$\therefore \text{ weight} = \frac{64 \times 8}{9} \text{ tons} = 56\tfrac{8}{9} \text{ tons.}$$

(2.) $1\frac{1}{5} : 240 = 3$ cwt. : weight supported ;

$$\therefore \text{ weight} = \frac{240 \times 3 \times 5}{6} \text{ cwt.} = 600 \text{ cwt.} = 30 \text{ tons.}$$

(3.) Area of small piston : area of large piston $= (1\frac{1}{8})^2 : (50)^2$

$$= 9 \times 9 : 50 \times 50 \times 64$$

$$\therefore \text{ weight supported } = \frac{50 \times 50 \times 64 \times 15}{9 \times 9} \text{ lbs.} = \frac{800000}{27} \text{ lbs.}$$

$$= 29629 \cdot \dot{6}2\dot{9} \text{ lbs.}$$

(4.) Let x represent the pressure in lbs.
Then $2 : 144 = x : 4\frac{1}{2}$;

$$\therefore x = \frac{2 \times 9}{144 \times 2} = \frac{1}{16}, \therefore \text{ pressure will be 1 ounce.}$$

(5.) Let x represent the pressure in lbs.
Then $1 : 144 = x : 9$;

$$\therefore x = \frac{9}{144} = \frac{1}{16}, \therefore \text{ pressure will be 1 ounce.}$$

(6.) Area of horizontal section of large cylinder $=\dfrac{10\times 10\times 22}{7}$ square inches.

Let x = measure of lifting power in cwts.

$$\text{Then } 1\tfrac{1}{8} : \frac{10\times10\times22}{7} = 20 : x\,;$$

$$\therefore\ x=\frac{352000}{63}\text{ cwt.}=5587\tfrac{19}{63}\text{ cwt.}$$

EXAMPLES—II. (p. 18.)

(1.) Pressure at $42\frac{2}{3}$ feet $=\dfrac{42\frac{2}{3}}{32}\cdot$ pressure at 32 feet

$$=\frac{128}{32\times3}\times 15\text{ lbs.}=20\text{ lbs.}$$

(2.) Pressure at $20\frac{1}{2}$ feet $=\dfrac{20\frac{1}{2}}{8}\cdot$ pressure at 8 feet

$$=\frac{41}{8\times2}\times\frac{44}{3}\text{ lbs.}=37\tfrac{7}{12}\text{ lbs.}$$

(3.) The pressure at a depth of 8 inches in the second fluid is double the pressure at a depth of 4 inches, and is therefore equal to the pressure at a depth of 6 inches in the first fluid.

$\therefore$ the pressures are as $7 : 6$.

(4.) The pressure at a depth of 12 inches in the second fluid is four times the pressure at a depth of 3 inches, and is therefore equal to the pressure at a depth of 8 inches in the first fluid.

$\therefore$ the pressures are as $9 : 8$.

(5.) Let x be the height of the column in feet. Then $(30-x)$ feet is the depth of the top of the column, and 30 feet is the depth of the bottom of the column.

$$\therefore\ 30-x : 30 = 2 : 3,$$

$$\text{or, } 90-3x=60,\text{ and }\therefore\ x=10\text{ feet.}$$

(6.) Pressure at 12 feet $=\frac{12}{15}$ pressure at 15 feet
$=\frac{12}{15}\times 15$ lbs. $=12$ lbs.

(7.) Pressure at 12 feet $=\frac{12}{4}$ pressure at 4 feet
$=\frac{12}{4}\times 3$ lbs. $=9$ lbs.

(8.) Cubic content of water $=(20 \times 2\frac{1}{2})$ cubic feet $=50$ cubic feet ;
$\therefore$ pressure $=(50 \times 1000)$ ounces $=3125$ lbs. $=1$ ton 7 cwt. 3 qrs. 17 lbs.

(9.) Cubic content of mercury $=(8 \times 3)$ cubic inches ;

$$\therefore \text{pressure} = \frac{13600 \times 24}{1728} \text{ ounces} = \frac{1700}{9} \text{ ounces} = 11 \text{ lbs. } 12\tfrac{8}{9} \text{ ounces.}$$

(10.) Cubic content of water $=(24 \times 15)$ cubic feet ;

$$\therefore \text{pressure} = (24 \times 15 \times 1000) \text{ ounces} = 22500 \text{ lbs.}$$

(11.) Let ABC represent the base of the cistern : AD a perpendicular on BC.

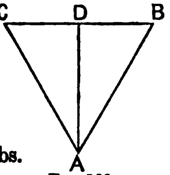

Fig. 188.

Then $AD = DB \,.\, \sqrt{3}$.

Hence, if each side of the triangle be 6 feet, area of base $= AD \,.\, BD = 9\sqrt{3}$;

$\therefore$ cubic content of water $=(9\sqrt{3} \times 2)$ cubic feet ;

$$\therefore \text{pressure on base} = \frac{9\sqrt{3} \times 2 \times 1000}{16} \text{ lbs.} = 1125\sqrt{3} \text{ lbs.}$$

(12.) When the spout has been broken midway, its upper extremity is three-fourths of the height of the tea-pot, and therefore this is the height to which the tea-pot can be filled.

(13.) Since the external pressure on the cork increases as the bottle sinks, while the internal pressure on the cork is constant, the cork will be forced in when the external pressure exceeds the internal pressure.

(14.) Pressure on bottom

$$= (112 \times 4 \times 1000) \text{ ounces} = \frac{112 \times 4 \times 1000}{16 \times 112 \times 20} \text{ tons} = 12\tfrac{1}{2} \text{ tons.}$$

(15.) Let x be the height of the water in the tube in inches.

The pressure produced by this on the base of the piston, which is a *square* inch in area, will be equal to the weight of x *cubic* inches of water, which has to lift 7 lbs. 13 ounces, or 125 ounces ;

$$\therefore \frac{x \times 1000}{1728} = 125 ;$$

$$\therefore x = \frac{1728 \times 125}{1000} = 216 \text{ inches} = 18 \text{ feet.}$$

Examples—III. (p. 27.)

(1.) 1 cubic foot of copper weighs $8 \cdot 91 \times 1000$ ounces, or, 8910 ounces.

1 cubic inch of copper weighs $\frac{8910}{1728}$ ounces.

512 cubic inches of copper weigh $\frac{8910 \times 512}{1728}$ ounces, or, 165 lbs.

(2.) 1 cubic inch of iron weighs as much as 18 cubic inches of amber.

$\therefore$ specific gravity of iron : specific gravity of amber $= 18 : 1$.

(3.) 1 cubic foot of mercury weighs 13500 ounces.

1 cubic inch of mercury weighs $\frac{13500}{1728}$ ounces, or, $7\frac{13}{16}$ ounces.

(4.) 1 cubic foot of the substance weighs 50 lbs.

1 cubic foot of the substance weighs 800 ounces,

and 1 cubic foot of water weighs 1000 ounces;

$\therefore$ specific gravity of the substance is $\cdot 8$.

(5.) 1 cubic foot of cork weighs $\cdot 24 \times 1000$ ounces $= 240$ ounces.

1 cubic inch of cork weighs $\frac{240}{1728}$ ounces.

36 cubic inches of cork weigh $\frac{200 \times 36}{1728}$ ounces, or, 5 ounces.

(6.) A cubic foot of the substance weighs 3000 ounces.

A cubic inch of the substance weighs $\frac{3000}{1728}$ ounces, or, $1\frac{53}{72}$ ounces.

(7.) 1 cubic foot of the body weighs $\frac{n}{m}$ lbs.

1 cubic foot of the body weighs $\frac{16n}{m}$ ounces,

and 1 cubic foot of water weighs 1000 ounces;

$$\therefore \text{specific gravity of body} : 1 = \frac{16n}{m} : 1000;$$

$$\therefore \text{specific gravity of body} = \frac{16n}{1000m} = \frac{\cdot 016n}{m}.$$

(8.) 1 cubic inch of iron weighs $4\frac{1}{2}$ ounces,

1 cubic foot of iron weighs $(4\frac{1}{2} \times 1728)$ ounces, or, 7776 ounces;

$\therefore$ specific gravity of iron : 1 = 7776 : 1000;

$\therefore$ specific gravity of iron = 7·776.

(9.) 1 cubic foot of the wood weighs $\frac{875 \times 16}{12}$ ounces, or, $1166{\cdot}\dot{6}$ ounces;

$\therefore$ specific gravity of the wood : 1 = $1166{\cdot}\dot{6}$: 1000;

$\therefore$ specific gravity of wood = $1{\cdot}1\dot{6}$.

(10.) 1 cubic foot of ash weighs $\frac{2743 \times 16}{26 \times 2}$ ounces, or, 844 ounces.

$\therefore$ specific gravity of ash : 1 = 844 : 1000;

$\therefore$ specific gravity of ash = ·844.

(11.) Let v be the volume of the metal, and s the specific gravity of the compound.

Then $\frac{v}{2}$ is the volume of the alloy;

$$\therefore v \times 15 + \frac{v}{2} \times 12 = \left(v + \frac{v}{2}\right) \times s;$$

$$\therefore 15 + 6 = \frac{3s}{2}, \text{ and } \therefore s = 14.$$

(12.) Let s be the specific gravity of the lump.

Then $\frac{4s}{3}$ is the specific gravity of one metal (for $2 : 1\frac{1}{2} = 4 : 3$),

and $\frac{4s}{5}$ is the specific gravity of the other (for $2 : 2\frac{1}{2} = 4 : 5$).

Let v_1 and v_2 be the volumes.

Then $v_1 \times \frac{4s}{3} + v_2 \times \frac{4s}{5} = (v_1 + v_2) \times s$;

$$\therefore 20v_1 + 12v_2 = 15v_1 + 15v_2;$$

$$\therefore 5v_1 = 3v_2, \text{ or, } v_1 : v_2 = 3 : 5.$$

Hence to form 2 cubic inches of the compound we must put $\frac{3}{4}$ cubic inches of first and $\frac{5}{4}$ cubic inches of second.

(13.) Let v_1 and v_2 be the volumes, w_1 and w_2 the weights.

Then $1{\cdot}5v_1 + 3v_2 = (v_1 + v_2)\,2{\cdot}5$,

$$\text{or, } 15v_1 + 30v_2 = 25v_1 + 25v_2;$$

$$\text{and } \therefore v_1 : v_2 = 5 : 10 = 1 : 2.$$

$$\text{Also, } \frac{w_1}{1{\cdot}5} + \frac{w_2}{3} = \frac{w_1 + w_2}{2{\cdot}5},$$

$$\text{or, } \frac{10w_1}{15} + \frac{w_2}{3} = \frac{10w_1 + 10w_2}{25},$$

$$\text{or, } 50w_1 + 25w_2 = 30w_1 + 30w_2;$$

$$\therefore 20w_1 = 5w_2;$$

$$\therefore w_1 : w_2 = 5 : 20 = 1 : 4.$$

(14.) Let x and y be the measures of sea-water and fresh water respectively.

Then $x \times 1{\cdot}027 + y \times 1 = (x + y) \times 1{\cdot}009$,

$$\text{or, } 1{\cdot}027x + y = 1{\cdot}009x + 1{\cdot}009y,$$

$$\text{or, } {\cdot}018x = {\cdot}009y;$$

$$\therefore x : y = 1 : 2;$$

that is, proportion of fresh water to be added $= 2 : 1$.

(15.) Let w be the measure of the weight of each substance, d the density of the compound.

$$\text{Then } \frac{w}{3{\cdot}25} + \frac{w}{2{\cdot}75} = \frac{2w}{d},$$

$$\text{or, } \frac{100}{325} + \frac{100}{275} = \frac{2}{d},$$

$$\text{or, } \frac{4}{13} + \frac{4}{11} = \frac{2}{d}, \therefore d = 2\tfrac{47}{48}.$$

(16.) Let v be the measure of the volume of each substance, s the specific gravity of the compound.

Then $v \times 2{\cdot}5 + v \times 1{\cdot}5 = 2v \times s$;

$$\therefore 2{\cdot}5 + 1{\cdot}5 = 2s, \text{ and } \therefore s = 2.$$

(17.) Let s be the specific gravity of the compound.

Then $5 \times 11{\cdot}35 + 5 \times 7{\cdot}3 = 10s$;

$$\text{or, } 11{\cdot}35 + 7{\cdot}3 = 2s, \text{ and } \therefore s = 9{\cdot}325.$$

(18.) Let v be the measure of the volume of each fluid, d_1, d_2, d_3 the measures of their densities, d the measure of the density of the mixture.

Then $vd_1 + vd_2 + vd_3 = (v + v + v)d$;

$$\therefore\ d_1 + d_2 + d_3 = 3d, \text{ and } \therefore\ d_3 = 3d - d_1 - d_2.$$

(19.) Let s be the specific gravity of the compound.

Then $10 \times 8{\cdot}9 + 7 \times 7{\cdot}3 = 17s$,

$$\text{or, } 89 + 51{\cdot}1 = 17s;$$

$$\therefore\ 17s = 140{\cdot}1, \text{ and } \therefore\ s = 8{\cdot}241 \ldots$$

(20.) Let s be the specific gravity of the mixture.

Then $\dfrac{5}{\cdot 7} + \dfrac{6}{\cdot 8} + \dfrac{7}{\cdot 9} = \dfrac{5 + 6 + 7}{s}$,

$$\text{or, } \frac{50}{7} + \frac{15}{2} + \frac{70}{9} = \frac{18}{s};$$

$$\therefore\ \frac{2825}{126} = \frac{18}{s}, \text{ and } \therefore\ s = {\cdot}802 \ldots$$

(21.) Let s be the specific gravity of standard gold.

Then $11 \times 19{\cdot}3 + 1 \times 8{\cdot}62 = 12s$;

$$\therefore\ 212{\cdot}3 + 8{\cdot}62 = 12s;$$

$$\therefore\ s = 18{\cdot}41.$$

(22.) Let s be the specific gravity of the mixture.

Then $63 \times 1{\cdot}82 + 24 \times 1 = 86s$,

$$\text{or, } 114{\cdot}66 + 24 = 86s;$$

$$\therefore\ s = 1{\cdot}61 \ldots$$

(23.) Let s be the specific gravity of the compound.

Then $4 \times 2 + 5 \times 3 + 6 \times 4 = (4 + 5 + 6)s$,

$$\text{or, } 8 + 15 + 24 = 15s;$$

$$\therefore\ s = 3{\cdot}1\dot{3}.$$

(24.) Let w be the weight of gold in ounces.

Then $\dfrac{w}{19{\cdot}35} + \dfrac{11{\cdot}5 - w}{2{\cdot}62} = \dfrac{11{\cdot}5}{7{\cdot}43}$,

$$\text{or, } \frac{100w}{1935} + \frac{1150 - 100w}{262} = \frac{1150}{743},$$

$$\text{or, } \frac{20w}{387} + \frac{575 - 50w}{131} = \frac{1150}{743};$$

$$\therefore\ \frac{2620w + 222525 - 19350w}{50697} = \frac{1150}{743},$$

$$\text{or, } \frac{222525 - 16730w}{50697} = \frac{1150}{743},$$

$$\text{or, } 165336075 - 12430390w = 58301550;$$

$$\therefore 107034525 = 12430390w,$$

$$\text{and } \therefore w = 8{\cdot}6 \ . \ . \ . \text{ ounces.}$$

(25.) Let w_1, w_2 be the weights of iron and gold, v_1, v_2 the volumes of iron and gold.

Then $7{\cdot}8v_1 + 19{\cdot}4v_2 = 8\ (v_1 + v_2)$,

$$\text{or, } 11{\cdot}4v_2 = {\cdot}2v_1;$$

$$\therefore v_1 : v_2 = 11{\cdot}4 : {\cdot}2$$

$$= 57 : 1.$$

$$\text{Again, } \frac{w_1}{7{\cdot}8} + \frac{w_2}{19{\cdot}4} = \frac{w_1 + w_2}{8},$$

$$\text{or, } \frac{5w_1}{39} + \frac{5w_2}{97} = \frac{w_1 + w_2}{8};$$

$$\therefore \frac{485w_1 + 195w_2}{3783} = \frac{w_1 + w_2}{8},$$

$$\text{or, } 3880w_1 + 1560w_2 = 3783w_1 + 3783w_2;$$

$$\therefore 97w_1 = 2223w_2;$$

$$\therefore w_1 : w_2 = 2223 : 97.$$

EXAMPLES—IV. (p. 42).

(1.) Weight of water displaced $= \frac{3}{10}$ of weight of glass;

$\therefore$ specific gravity of water $= \frac{3}{10}$ of specific gravity of glass;

$\therefore$ specific gravity of glass $= \frac{10}{3} = 3{\cdot}\dot{3}$.

(2.) Pressure

$= \text{weight of } \{(28 \times 1760 \times 3) \times \frac{1}{4} \times 480\}$ cubic feet of sea-water.

$$= \frac{28 \times 1760 \times 3 \times 480 \times 1{\cdot}026 \times 1000}{4} \text{ ounces}$$

$$= \frac{7 \times 1760 \times 3 \times 480 \times 1026}{16 \times 112 \times 20} \text{ tons.}$$

$= (110 \times 9 \times 513)$ tons $= 507870$ tons.

(3.) Let x = the part immersed, v = volume of body.
Then $x : v = 3{\cdot}3 : 4{\cdot}4$;

$$\therefore\ x = \frac{3{\cdot}3}{4{\cdot}4}v = \frac{33}{44}v = \frac{3}{4} \text{ of whole body.}$$

(4.) Let x be the number of grains a sovereign weighs in water.
Then $122{\cdot}5 - x$ = weight of water displaced;

$$\therefore\ 19{\cdot}4 : 1 = 122{\cdot}5 : 122{\cdot}5 - x,$$

$$\text{or, } 2376{\cdot}5 - 19{\cdot}4x = 122{\cdot}5;$$

$$\therefore\ x = \frac{2376{\cdot}5 - 122{\cdot}5}{19{\cdot}4} = 116\tfrac{18}{97} \text{ grains} = 4 \text{ dwt. } 20\tfrac{18}{97} \text{ grains.}$$

(5.) Weight of water displaced = 2 lbs.;

$$\therefore \text{ specific gravity of substance} : 1 = 8 : 2;$$

$$\therefore \text{ specific gravity of substance} = 4.$$

(6.) Let x be the weight required, w the weight of the tub.
Then since weight of water displaced by the whole tub $= 4w$,

$$x = 4w - w = 3w = \text{ three times weight of the tub.}$$

(7.) Part immersed : whole body $= 1{\cdot}4 : 2{\cdot}1$;

$$\therefore \text{ part immersed} = \frac{14}{21} \text{ of whole body} = \frac{2}{3} \text{ of body.}$$

(8.) Weight of water displaced by lead $= \dfrac{1}{11{\cdot}4}$ ounces;

$$\therefore \text{ pressure on bottom} = \left(1 - \frac{1}{11{\cdot}4}\right) \text{ ounces} = \frac{10{\cdot}4}{11{\cdot}4} \text{ ounces} = \frac{52}{57} \text{ ounces.}$$

(9.) Half a cubic inch of water weighs $\dfrac{500}{1728}$ ounces,

$$\text{a cubic inch of cork weighs } \frac{240}{1728} \text{ ounces};$$

$$\therefore \text{ weight to be added} = \frac{500 - 240}{1728} \text{ ounces} = \frac{65}{432} \text{ ounces.}$$

(10.) Since the specific gravity of the cork is one-fourth of the specific gravity of water, weight of water displaced by the cork = four times weight of the cork.

$\therefore$ tension of string = three times weight of the cork = 3 ounces.

(11.) Weight of water displaced by lead $=\frac{46}{11\cdot5}$ ounces;

$\therefore$ tension of string $=\left(46-\frac{46}{11\cdot5}\right)$ ounces $=\left(46\times\frac{11\cdot5-1}{11\cdot5}\right)$ ounces

$=\frac{46\times105}{115}$ ounces $=42$ ounces.

(12.) Weight of the wood $=\left(\cdot24\times\frac{1000}{1728}\right)$ ounces $=\frac{240}{1728}$ ounces;

$\therefore$ we must add a weight of $\frac{240}{1728}$ ounces, or, $\frac{5}{36}$ ounces.

(13.) Weight of water displaced $=\frac{1}{3}$ of half the weight of the cube

$=\frac{1}{6}$ of weight of the cube;

$\therefore$ tension of string $=\frac{5}{6}$ weight of cube $=\frac{5}{6}$ lbs.

(14.) Weight of water displaced $=(42-30)$ ounces $=12$ ounces;

$\therefore$ specific gravity of substance : $1=42:12$;

$\therefore$ specific gravity of substance $=\frac{42}{12}=3\cdot5$.

(15.) 14 lbs. $=(14\times16)$ ounces $=224$ ounces.

$\therefore$ weight of water displaced $=(2560-224)$ ounces $=2336$ ounces;

$\therefore$ specific gravity of substance : $1=2560:2336$;

$\therefore$ specific gravity of substance $=\frac{2560}{2336}=\frac{80}{73}$.

(16.) Weight of water displaced by substance

$=(20+12-18)$ ounces $=14$ ounces;

$\therefore$ specific gravity of substance : $1=12:14$;

$\therefore$ specific gravity of substance $=\frac{12}{14}=\frac{6}{7}$.

(17.) Weight of water displaced by mahogany

$=(375+380-300)$ grains $=455$ grains;

$\therefore$ specific gravity of mahogany : $1=375:455$;

$\therefore$ specific gravity of mahogany $=\frac{375}{455}=\frac{75}{91}$.

(18.) Weight of water displaced by metal

$$=(120-113) \text{ grains}=7 \text{ grains};$$

$$\therefore \text{ specific gravity of metal} : 1=120 : 7;$$

$$\therefore \text{ specific gravity of metal}=\frac{120}{7}=17\tfrac{1}{7}.$$

(19.) Weight of water displaced by spar

$$=(190-120) \text{ grains}=70 \text{ grains};$$

$$\therefore \text{ specific gravity of spar} : 1=190 : 70;$$

$$\therefore \text{ specific gravity of spar}=\frac{190}{70}=\frac{19}{7}=2\tfrac{5}{7}.$$

(20.) Weight of water displaced by former body

$$=(4+3-2\tfrac{1}{4}) \text{ ounces}=4\tfrac{3}{4} \text{ ounces};$$

$$\therefore \text{ specific gravity of former body} : 1=4 : 4\tfrac{3}{4};$$

$$\therefore \text{ specific gravity of former body}=\frac{4}{4\frac{3}{4}}=\frac{16}{19}.$$

(21.) Weight of water displaced by wood and lead

$$=(22+12-8) \text{ lbs.}=26 \text{ lbs.}$$

Weight of water displaced by lead

$$=\frac{22}{11{\cdot}35} \text{ lbs.}=\frac{2200}{1135} \text{ lbs.}=\frac{440}{227} \text{ lbs.};$$

$$\therefore \text{ weight of water displaced by wood}=\left(26-\frac{440}{227}\right) \text{ lbs.}=\frac{5462}{227} \text{ lbs.};$$

$$\therefore \text{ specific gravity of wood} : 1=12 : \frac{5462}{227};$$

$$\therefore \text{ specific gravity of wood}=\frac{12\times 227}{5462}=\frac{1362}{2731}.$$

(22.) Let w=weight of substance *in vacuo*, $2w$=weight of sinker *in vacuo*, s=specific gravity of substance;

$$\therefore (w+2w)-2w, \text{ or, } w=\text{weight lost by both in water.}$$

Let v=volume of sinker, then $v\times 1$=weight of water displaced by sinker;

$$\therefore w-v=\text{weight lost by substance};$$

$$\therefore \frac{w-v}{w}=\frac{1}{s},$$

$$\text{or, } \frac{vs-v}{vs}=\frac{1}{s}, \text{ and } \therefore \frac{s-1}{s}=\frac{1}{s}, \text{ or } s=2.$$

(23.) A cubic foot of cork weighs $(\cdot 24 \times 1000)$ ounces, or 240 ounces; $\therefore$ to displace a cubic foot of water, weighing 1000 ounces, we must increase the weight of the cork by 760 ounces, or $47\frac{1}{2}$ lbs.

(24.) Let x = volume of part immersed.

Then $x : x+8 = 1 : 3$;

$$\therefore 3x = x+8;$$
$$\therefore x = 4.$$

Hence whole length of the cylinder = 12 feet.

(25.) Four-fifths of the cylinder are immersed.

Let s be the specific gravity of the cylinder.

Then $s : \cdot 825 = \frac{4}{5} : 1$;

$$\therefore s = \frac{4 \times \cdot 825}{5} = 4 \times \cdot 165 = \cdot 66.$$

(26.) Because the specific gravity of salt water is greater than that of fresh water.

(27.) Let x parts of an inch be in the mercury, then $1-x$ parts of an inch are in the water.

Then $x \times 13\cdot 6 + (1-x) \times 1 = 1 \times 7\cdot 8$,

$$\text{or } 13\cdot 6x + 1 - x = 7\cdot 8;$$
$$\therefore 12\cdot 6x = 6\cdot 8;$$
$$\therefore x = \frac{68}{126} = \frac{34}{63} \text{ inches.}$$

(28.) Since the specific gravity of the body is half the specific gravity of water, it will float with 500 ounces of its weight above the surface.

And since 1 cubic foot of water weighs 1000 ounces,

1 cubic foot of the body weighs 500 ounces;

$\therefore$ 1728 cubic inches of the body are above the surface.

(29.) 12 cubic inches of water weigh $\frac{12 \times 1000}{1728}$ ounces, or, $\frac{125}{18}$ ounces, or, $6\frac{17}{18}$ ounces;

$\therefore$ weight of body in water $=$ 8 lbs. $- 6\frac{17}{18}$ ounces, or, 7 lbs. $9\frac{1}{18}$ ounces.

(30.) Let x = edge of the cube in inches ;

$\therefore\ 1 \times x^2$ = cubic inches of water displaced by the cube sinking an inch ;

$$\therefore\ x^2 \times \frac{1000}{1728} = \text{weight of this water in ounces};$$

$$\therefore\ x^2 \times \frac{1000}{1728} = \frac{1000}{3};$$

$$\therefore\ x^2 = 576, \text{ and } x = 24 \text{ inches} = 2 \text{ feet}.$$

(31.) Content of box *externally* $= (10 \times 8 \times 6)$ cubic inches $= 480$ cubic inches.

Content of box internally $= (9\frac{1}{2} \times 7\frac{1}{2} \times 5\frac{1}{2})$ cubic inches $= \frac{3135}{8}$ cubic inches.

$\therefore$ content of substance $= \left(480 - \frac{3135}{8}\right)$ cubic inches $= \frac{705}{8}$ cubic inches.

Hence, if s be the specific gravity of substance,

$$\frac{705s}{8} = 480 \times 1, \text{ or, } s = \frac{480 \times 8}{705} = \frac{3840}{705};$$

$$\therefore\ s = 5\tfrac{21}{47}.$$

(32.) Let DE be the surface of the fluid.

Then $ADE : AFG = 1 : 3$.

(HYDROSTATICS, Art. 63.)

But $ADE : AFG = AB^2 : AC^2$. (EUCLID, VI. XIX.)

$$\therefore\ AB^2 : AC^2 = 1 : 3;$$

$$\therefore\ AB = \frac{AC}{\sqrt{3}}.$$

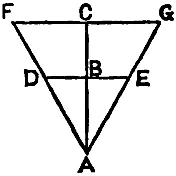

FIG. 189.

(33.) DE being the surface of the fluid, and D, E the middle points of AB, AC, triangle ADE = one-fourth of triangle ABC.

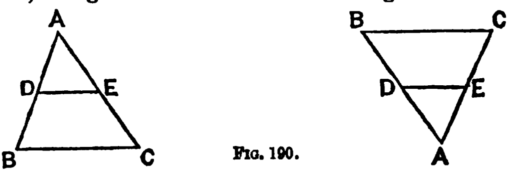

FIG. 190.

Now, part immersed : whole = specific gravity of body : specific gravity of fluid.

∴ when the vertex is downwards, specific gravity of lamina = one-fourth specific gravity of fluid; and when the vertex is upwards, specific gravity of lamina = three-fourths specific gravity of fluid.

(34.) Let x be the number of pounds in the weight, s the specific gravity of the fluid, v and $\frac{v}{3}$ the volumes of fluid displaced.

$$\text{Then } 8+x=v\,.\,s,$$

$$\text{and } 8=\frac{v}{3}\,.\,s\,;$$

$$\therefore \frac{8+x}{8}=3, \text{ or } x=16 \text{ lbs.}$$

(35.) Let s be the specific gravity of the cylinder.

Then $\frac{3}{4}:1=s:1$;

$$\therefore\ s=\frac{3}{4}=\cdot 75.$$

(36.) Volume of cube $=\left(\frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}\right)$ cubic feet $=\frac{27}{8}$ cubic feet;

∴ $\frac{27}{8}$ cubic feet of the material weigh 2250 ounces;

∴ 1 cubic foot of the material weighs $\frac{2000}{3}$ ounces;

∴ specific gravity of the material is $\frac{2}{3}$.

Then if x be the distance in inches to which the cylinder sinks,

$$x:3=\frac{2}{3}:1,$$

$$\text{and } \therefore\ x=2 \text{ inches.}$$

(37.) Weight of fluid displaced = 1 lb.

Then, if s be the specific gravity of the fluid,

$$1:3=s:2\cdot 7\,;$$

$$\therefore\ 3s=2\cdot 7, \text{ or } s=\cdot 9.$$

(38.) Let v be the volume of the part immersed in cubic inches.

Then $v:1=2\cdot 6:13\cdot 5$;

$$\therefore\ v=\frac{2\cdot 6}{13\cdot 5}=\frac{26}{135}$$

∴ the aluminium will sink to a depth of $\frac{26}{135}$ inch.

(39.) Let w be the weight of the body in pounds.

Then $w : w+2 = \frac{2}{3} : 1$;

$$\therefore w = \frac{2w+4}{3}, \text{ or, } w = 4 \text{ lbs.}$$

(40.) Let p be the pressure in pounds.

Then $3 : 3+p = \frac{1}{2} : 1$,

$$\text{or } 3 = \frac{3+p}{2}, \text{ or, } p = 3 \text{ lbs.}$$

(41.) 2 cubic feet of water weigh 2000 ounces.
2 cubic feet of cork weigh 480 ounces.

$\therefore$ tension of string $= (2000 - 480)$ ounces $= 1520$ ounces $= 95$ lbs.

(42.) Let s_1 and s_2 be the specific gravity of the bodies, s the specific gravity of the fluid.

$$\text{Then } \frac{w_1}{w_1 - w} = \frac{s_1}{s},$$

$$\text{and } \frac{w_2}{w_2 - w} = \frac{s_2}{s};$$

$$\therefore \frac{w_1(w_2 - w)}{w_2(w_1 - w)} = \frac{s_1}{s_2}.$$

(43.) If the specific gravity of the wood be less than the specific gravity of the water, the water filling up the cavity will cause an increase in the apparent weight of the bullet.

(44.) Weight of first fluid displaced $= 3$ lbs., weight of second fluid displaced $= 2$ lbs.

If then s_1 and s_2 be the specific gravity of the two fluids, and s the specific gravity of the body,

$$\frac{s_1}{s} = \frac{3}{6}, \text{ and } \frac{s_2}{s} = \frac{2}{6};$$

$$\therefore \text{dividing, } \frac{s_1}{s_2} = \frac{3}{2}.$$

(45.) Weight of fluid displaced by the body $= 1$ lb.;

$\therefore$ if s be the specific gravity of the fluid,

$$\frac{s}{7{\cdot}7} = \frac{1}{7}, \text{ and } \therefore s = \frac{7{\cdot}7}{7} = 1{\cdot}1.$$

(46.) Let s_1 be the specific gravity of the first sphere, and s_2 be the specific gravity of the second sphere.

Then $4s_1$ is the specific gravity of the fluid.

Let $2v$ be the volume of the first sphere.

Then $3v$ is the volume of the second sphere,

$$\text{and } 2v \,.\, s_1 + 3v \,.\, s_2 = (2v+3v) \,.\, 4s_1 ;$$

$$\therefore\ 2s_1 + 3s_2 = 20s_1 ;$$

$$\therefore\ s_2 = 6s_1.$$

(47.) Let w be the weight in grains of the copper in air.

$$\text{Then } \frac{w-887}{w} = \frac{1}{8\cdot 85} ;$$

$$\therefore\ 885w - 887 \times 885 = 100w ;$$

$$\therefore\ 785w = 887 \times 885 ;$$

$$\therefore\ w = \frac{156999}{157}.$$

Then if s be the specific gravity of the alcohol,

$$s : 1 = \frac{156999}{157} - 910 : \frac{156999}{157} - 887 \quad \text{(Hydrostatics, Art. 68.)}$$

$$\therefore\ s = \frac{156999 - 142870}{156999 - 139259} = \frac{14129}{17740} = \cdot 8 \text{ nearly.}$$

(48.) Let x be the depth in inches to which it sinks in alcohol.

Then $x \times \cdot 79 = 4 \times 1$;

$$\therefore\ x = \frac{4}{\cdot 79} = \frac{400}{79} = 5\tfrac{5}{79} \text{ inches.}$$

(49.) Five lbs. of the compound are immersed. Let x be the number of lbs. of silver in the part immersed, $5-x$ the number of lbs. of aluminium in the part immersed.

Then since 10 lbs. of mercury are displaced

$$\frac{x}{10\cdot 4} + \frac{5-x}{2\cdot 6} = \frac{10}{13\cdot 5},$$

$$\text{or, } \frac{x}{104} + \frac{5-x}{26} = \frac{10}{135} ;$$

$$\therefore\ \frac{20-3x}{104} = \frac{2}{27} ;$$

$$\therefore\ 540 - 81x = 208, \text{ or, } x = 4\tfrac{8}{81} \text{ lbs.} ;$$

$$\therefore \text{ there are } 8\tfrac{16}{81} \text{ lbs. of silver in the whole mass.}$$

(50.) Let w = weight in pounds of water displaced by the rod.
Then $x : 5 = 1 : 7{\cdot}8$;

$$\therefore\ x = \frac{50}{78} = \frac{25}{39}.$$

Hence tension of string $= \left(10 - \frac{25}{39}\right)$ lbs. $= 9\frac{14}{39}$ lbs.

(51.) Weight of water displaced $= (1 - {\cdot}905)$ ounces $= {\cdot}095$ ounces;
$\therefore$ if s be the specific gravity of the silver,

$$\frac{s}{1} = \frac{1}{{\cdot}095} = \frac{1000}{95};$$

$$\therefore\ s = 10\tfrac{10}{19}.$$

(52.) Weight of water displaced by first body $= \frac{1}{2}$ lb., and, if s be the specific gravity of the second body, weight of water displaced by second body $= \frac{2}{s}$ lbs.;

$\therefore$ tension of first string $= \frac{1}{2}$ lb.,
and tension of second string $= \left(2 - \frac{2}{s}\right)$ lbs.

Now these tensions must be equal;

$$\therefore\ \frac{1}{2} = 2 - \frac{2}{s},\ \text{and}\ \therefore\ s = \frac{4}{3} = 1{\cdot}\dot{3}.$$

(53.) Let x inches be above the surface.
Then $9 - x : 9 = \frac{1}{3} : 1$;

$$\therefore\ 9 - x = 3,\ \text{and}\ \therefore\ x = 6\ \text{inches}.$$

(54.) If S and S' be the specific gravity of the fluids, V and V' the parts immersed.

$$S : S' = V' : V. \qquad \text{(Hydrostatics, Art. 66.)}$$
$$= m - y : m - x.$$

(55.) The weight of fluid displaced by body and lead $= 7$ lbs.
Now $5\frac{1}{2}$ lbs. of lead displace $\frac{1}{2}$ lb. of water;
$\therefore$ $5\frac{1}{2}$ lbs. of lead displace 2 lbs. of the fluid;
$\therefore$ weight of fluid displaced by body $= 5$ lbs.
Then if s be the specific gravity of the body,

$$s : 4 = 3 : 5;$$

$$\therefore\ s = \frac{12}{5} = 2\tfrac{2}{5}.$$

(56.) Let w = weight of the substance in air in ounces.

Then $\frac{w-10}{w-15}=\frac{\text{specific gravity of water}}{\text{specific gravity of alcohol}}=\frac{1}{\cdot 7947}$.

(HYDROSTATICS, Art. 68.)

$$\therefore \cdot 7947w - 7\cdot 947 = w - 15;$$

$$\therefore 7\cdot 053 = \cdot 2053w, \text{ or, } w = \frac{70530}{2053} \text{ ounces.}$$

Hence, weight of water displaced by substance

$$= \left(\frac{70530}{2053} - 10\right) \text{ ounces}$$

$$= \frac{50000}{2053} \text{ ounces};$$

$\therefore$ if x be the number of cubic inches in the substance,

$$\frac{1000x}{1728} = \frac{50000}{2053};$$

$$\therefore x = \frac{50 \times 1728}{2053} = \frac{86400}{2053} \text{ cubic inches.}$$

(57.) Let x cubic yards be the volume of the granite, $(1-x)$ cubic yards is the volume of the ice.

Then $x \times 2\cdot 65 + (1-x) \times \cdot 918 = \frac{23}{25} \times 1$;

$$\text{or, } 2\cdot 65x + \cdot 918 - \cdot 918x = \cdot 92;$$

$$\therefore 1\cdot 732x = \cdot 002;$$

$$\therefore x = \frac{1}{866} \text{ cubic yard.}$$

(58.) 320 ounces = 20 lbs.;

$\therefore$ specific gravity of the wood : specific gravity of water = 20 : 12.

Hence if x be the part of the wood below the surface,

$$x : 1 = 12 : 20;$$

$$\therefore x = \frac{12}{20} = \frac{3}{5}.$$

(59.) 3 inches of the wood weigh as much as $2\frac{1}{4}$ inches of water;

$\therefore$ 1 inch of the wood weighs as much as $\frac{3}{4}$ inch of water;

$\therefore$ 1728 inches of the wood weigh $\frac{3 \times 1000}{4}$ ounces, or, 750 ounces.

(60.) Let x parts of a cubic foot be immersed.
Then $x : 1 = \cdot 85 : 1\cdot 03$;

$$\therefore x = \frac{\cdot 85}{1\cdot 03} = \frac{85}{103}.$$

(61.) Let x be the number of feet in an edge of the cube.
Then $x^2 (x - 100) : x^3 = \cdot 9214 : 1\cdot 0263$;

$$\therefore 1\cdot 0263x - 102\cdot 63 = \cdot 9214x;$$

$$\therefore \cdot 1049x = 102\cdot 63;$$

$$\therefore x = \frac{1026300}{1049} = 978\cdot 3 \text{ nearly.}$$

$$\therefore x^3 = 936302451\cdot 687 \text{ nearly.}$$

(62.) Let x represent the pressure.
Then $w + x : w = 4 : 3$;

$$\therefore 3w + 3x = 4w, \text{ and } \therefore x = \frac{w}{3}.$$

(63.) The volume of fluid, v, displaced is the same in each case. Hence, if s and s' be the specific gravity of the fluids respectively

$$vs : vs' = m : n;$$

$$\text{and } \therefore s : s' = m : n.$$

(64.) Let v be the volume in inches of the part immersed in water.
Then $v : v + 4 = 1 : 3$;

$$\therefore 3v = v + 4, \text{ or } v = 2.$$

Hence, if x be the weight of the hydrometer in grains,

$$x + 40 : x = 2 : 2 - \frac{1}{12};$$

$$\text{or } (x + 40) \times 23 = 24x;$$

$$\therefore x = 920 \text{ grains.}$$

(65.) 980 divisions are below the surface in distilled water, 954 divisions are below the surface in sea water;

$\therefore$ if s be the specific gravity of sea water,

$$s : 1 = 980 : 954;$$

$$\therefore s = \frac{980}{954} = \frac{490}{477} = 1\tfrac{13}{477} = 1\cdot 0272 \ldots$$

(66.) Weight of water displaced by wood $=(13+6-8)$ lbs. $=11$ lbs.;

$\therefore$ if s be the specific gravity of the wood,

$$s:1=6:11;$$

$$\therefore s=\frac{6}{11}=\cdot 54.$$

(67.) Let x be the weight of the hydrometer in grains.

Then $\dfrac{x+67}{x}=\dfrac{1}{\cdot 866}$;

$$\therefore \cdot 866x+58\cdot 022=x;$$

$$\therefore \cdot 134x=58\cdot 022;$$

$$\therefore x=\frac{58022}{134}=433 \text{ grains.}$$

(68.) B weighs in water $(11-10)$ ounces, or, 1 ounce;

B weighs in air 15 ounces;

$\therefore$ if v be the volume and s the specific gravity of B,

$$\frac{v(s-\cdot 0013)}{v(s-1)}=\frac{15}{1};$$

$$\therefore s-\cdot 0013=15s-15.$$

$$\therefore 14s=14\cdot 9987;$$

$$\therefore s=1\cdot 0713 \text{ nearly.}$$

(69.) Let w = weight of the substance in air.

Then $\dfrac{w-20}{w-25}=\dfrac{\text{specific gravity of water}}{\text{specific gravity of alcohol}}=\dfrac{1}{\cdot 7947}$;

(HYDROSTATICS, Art. 68.)

$$\therefore \cdot 7947w-15\cdot 894=w-25;$$

$$\therefore 9\cdot 106=\cdot 2053w, \text{ and } \therefore w=\left(\frac{91060}{2053}-20\right) \text{ ounces.}$$

Hence weight of water displaced by substance

$$=\left(\frac{91060}{2053}-20\right) \text{ ounces}$$

$$=\frac{50000}{2053} \text{ ounces};$$

$\therefore$ if x be the number of cubic inches in the substance

$$\frac{1000x}{1728}=\frac{50000}{2053};$$

$$\therefore x=\frac{86400}{2053} \text{ cubic inches}=42\tfrac{174}{2053} \text{ cubic inches.}$$

EXAMPLES—V. (p. 61.)

(1.) Let the measure of the capacity of the barrel be x.
Then the measure of the capacity of the receiver is $10x$;

$$\therefore \text{density after sixth stroke} = \left(\frac{10x}{10x+x}\right)^6 . \text{ original density}$$
$$= \left(\frac{10}{11}\right)^6 \cdot \text{ original density.}$$

(2.) As in Example (1.)

$$\text{density after two strokes} = \left(\frac{8x}{8x+x}\right)^2 . \text{ original density}$$
$$= \frac{64}{81} \text{ times original density.}$$

(3.) Let x and y be the capacities of the receiver and the barrel.
Then $\left(\frac{x}{x+y}\right)^3 = \frac{729}{1000}$;

$$\therefore \frac{x}{x+y} = \frac{9}{10};$$
$$\therefore 10x = 9x + 9y, \text{ and } \therefore x : y = 9 : 1.$$

(4.) No: because the pressure varies with the depth alone; so that if the section varied there would still be equal vertical increments of space for equal increments of pressure.

(5.) Pressure on a square inch = weight of a column of water whose height is 90 feet and base a square inch + weight of column of mercury whose height is 30 inches and base a square inch.

$$= \text{weight of } \left(90 \times \frac{1}{144}\right) \text{ cubic feet of water + weight of 30 cubic inches of mercury}$$

$$= \frac{90 \times 1000}{144} \text{ ounces} + \frac{30 \times 47}{6} \text{ ounces} = (625 + 235) \text{ ounces} = 53\tfrac{3}{4} \text{ lbs.}$$

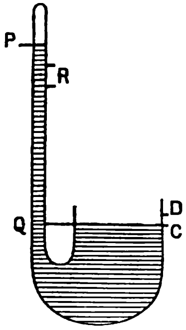

FIG. 191.

(6.) When the mercury stands at P in the tube, let C be the point at which it stands in the basin.

Now let the mercury fall to R in the tube, and rise to D in the basin.

$$\text{Then } CD = \frac{1}{10}PR;$$

$$\therefore \text{ real fall} = \text{apparent fall} + \frac{1}{10} \text{ of apparent fall}$$

$$= 1\tfrac{1}{2} \text{ inches} + \frac{1}{10} \text{ of } 1\tfrac{1}{2} \text{ inches}$$

$$= \left(\frac{3}{2} + \frac{3}{20}\right) \text{ inches} = 1\tfrac{13}{20} \text{ inches.}$$

(7.) The mercury would fall to the level of the mercury in the basin if the hole were made *above* the column of mercury in the tube. If it were made *in* the part occupied by the mercury, it will drive part of the column up the tube, and the other part down.

(8.) Pressure on $\frac{1}{234}$ of a square inch $=$ 1 ounce;

$\therefore$ pressure on a square inch $=$ 234 ounces $=$ 14·625 lbs.

(9.) Let x be the height of the column in inches.

Then $x \times 3{\cdot}4 = 30 \times 13{\cdot}6$;

$$\therefore x = \frac{30 \times 136}{34} = 120 \text{ inches} = 10 \text{ feet.}$$

(10.) It will have no effect, because a volume of mercury equal to that displaced by the iron will descend and allow the iron to take its place without disturbing the general upper surface.

(11.) The body floats in two fluids, air and water, and the absolute weight of the floating body is equal to the weight of the displaced fluids; hence if the air be removed, more water must be displaced to support the body, and therefore the body sinks.

(12.) If the air be admitted above the column of mercury it will drive it down a little, the extent of the fall depending on the quantity of air admitted.

(13.) A column of water 20 feet high represents a pressure of $\frac{20}{34}$ of 15 lbs. on the square inch.

$\therefore$ if x be the area of the valve in square inches,

$$\frac{20 \times 15 \times x}{34} \text{ lbs.} = \text{pressure on valve} = 21 \text{ lbs.};$$

$$\therefore x = \frac{34 \times 21}{20 \times 15} = \frac{714}{300} = 2{\cdot}38 \text{ square inches.}$$

(14.) 500 fathoms = $(500 \times 6 \times 12)$ inches = 36000 inches ;

$\therefore$ if d_1 be the density of atmospheric air and d_2 be the density of compressed air,

$$d_2 : d_1 = (30 \times 13{\cdot}57) : (36000 \times 1{\cdot}027) + (30 \times 13{\cdot}57)$$
$$= 407{\cdot}1 : 37379{\cdot}1 \,;$$
$$\therefore d_2 = \frac{4071}{373791} = {\cdot}0109 \text{ nearly.}$$

(15.) The water barometer under the same additional pressure would rise (20×12) inches, or 240 inches ;

$\therefore$ the mercurial barometer will rise $\frac{240}{13{\cdot}57}$ inches, or $\frac{24000}{1357}$ inches,

or, $17\frac{931}{1357}$ inches, or 1 foot $5\frac{931}{1357}$ inches.

(16.) When the floating body is partially immersed, both air and water are displaced ; but the *absolute* weight of floating body = weight of displaced fluids, which must therefore be constant. Therefore when the barometer rises, there must be a smaller water displacement, that is, the body rises ; while a decrease in the atmospheric pressure, when the barometer falls, will necessitate an increased water displacement, and therefore the body will sink a little.

(17.) Pressure at surface = the atmospheric pressure.

Pressure at 15 inches below the surface of mercury =

(atmospheric pressure at surface of water)
+ (pressure at depth of 17 feet of water)
+ (pressure at depth of 15 inches of mercury)
= (atmospheric pressure) + $\frac{1}{2}$ (atmospheric pressure)
+ $\frac{1}{2}$ (atmospheric pressure)
= twice atmospheric pressure ;

$\therefore$ the pressures are as 1 : 2.

(18.) A balloon is a hollow envelope, made of some light material, as silk, which, when filled with heated air, or some gas, rises in the air. This must be the result, if the weight of the air displaced by the balloon exceeds the weight of the balloon.

(19.) The external pressure on the bladder, increasing as the bladder is forced lower into the water, compresses the bladder, till at last the volume occupied by the air in the bladder is less than a volume of water equal to the weight of the bladder and air contained therein, and then the bladder must sink.

(20.) Let x be the height of the column in inches.

Then pressure on a square inch

$$= \text{weight of } x \text{ cubic inches of mercury}$$
$$= \text{weight of } 13{\cdot}56x \text{ cubic inches of water}$$
$$= \frac{13{\cdot}56x \times 1000}{1728} \text{ ounces.}$$
$$\therefore \frac{13{\cdot}56x \times 1000}{1728} = 226;$$
$$\therefore x = \frac{226 \times 144}{1130} = \frac{32544}{1130} = 28{\cdot}8 \text{ inches.}$$

(21.) Area of piston $= 36$ square inches.

Original pressure on piston $= (36 \times 15)$ lbs. $= 540$ lbs.

Then since the pressure of air varies inversely as the space it occupies,

new upward pressure on piston : $540 = 216$ cubic inches : 72 cubic inches

$$= 3 : 1;$$

$\therefore$ upward pressure on piston $= 1620$ lbs,

and downward pressure on piston by atmosphere $= 540$ lbs.;

$\therefore$ a weight of 1080 lbs. must be placed on it.

(22.) Height of water barometer, when the pump ceases to work, is 364 inches;

$$\therefore \text{ height of mercurial barometer} = \frac{364}{13{\cdot}6} \text{ inches} = \frac{3640}{136} \text{ inches}$$
$$= 26\tfrac{13}{17} \text{ inches.}$$

(23.) Height of column of mercury within the bottle = twice the height of mercurial column exposed to the ordinary pressure of the atmosphere = 60 inches = 5 feet.

(24.) Let x and y be the capacities of the receiver and the barrel.

Then $\dfrac{x+3y}{x}=\dfrac{3}{2}$;

$$\therefore\ 2x+6y=3x, \text{ and } \therefore\ x:y=6:1.$$

(25.) The air will be compressed inside, and so will displace less water; and since the cylinder floated before the increase of atmospheric pressure, it will now sink, because the weight of fluid displaced is now less than the weight of the cylinder.

(26.) Let x and y be the capacities of the receiver and the cylinder.

$$\begin{aligned}\text{Pressure after twenty strokes} &= \frac{x+20y}{x}\cdot \text{(original pressure)}\\ &= \frac{5y+20y}{5y} \text{ (original pressure)}\\ &= \text{five times original pressure.}\end{aligned}$$

(27.) Let x be the depth in feet.

Then $31:30+x=1:2$,

or, $62=30+x$, and $\therefore\ x=32$ feet.

(28.) Area of base of cube = 100 square inches,

$$\text{and pressure on base} = \left(1000\times\frac{1000}{1728}\right) \text{ ounces};$$

$$\begin{aligned}\therefore \text{ pressure on a square inch of base} &= \left(10\times\frac{1000}{1728}\right) \text{ ounces}\\ &= 5\tfrac{85}{108} \text{ ounces.}\end{aligned}$$

(29.) Pressure on 16 square inches of surface = (236×16) ounces = 3776 ounces.

Weight of column of water of $(16\times 9\times 12)$ cubic inches

$$=\left(16\times 9\times 12\times\frac{1000}{1728}\right) \text{ ounces} = 1000 \text{ ounces};$$

$$\therefore \text{ whole pressure} = 4776 \text{ ounces.}$$

(30.) Let D be a point in the surface of the fluid, and DA a vertical line.

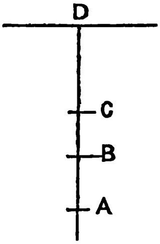

FIG. 192.

Then $\dfrac{\text{pressure at } B}{\text{pressure at } A}=\dfrac{DB}{DA}$;

$$\therefore \frac{\text{pressure at } A-\text{pressure at } B}{\text{pressure at } A}=\frac{DA-DB}{DA}=\frac{AB}{DA}.$$

Similarly,

$$\frac{\text{pressure at } A-\text{pressure at } C}{\text{pressure at } A}=\frac{DA-DC}{DA}=\frac{AC}{DA}.$$

By division,

$$\frac{\text{pressure at } A-\text{pressure at } B}{\text{pressure at } A-\text{pressure at } C}=\frac{AB}{AC};$$

$$\therefore \frac{p}{q}=\frac{AB}{AC};$$

$$\therefore \frac{p}{q-p}=\frac{AB}{AC-AB}=\frac{AB}{BC}.$$

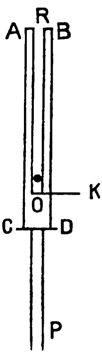

FIG. 193.

(31.) An outer barrel $ACDB$ encloses an inner barrel OR into which the bullet is rammed.

Air is injected through the end of the outer barrel through the stock P. When a quantity of air has been confined in the barrel $ACDB$, a trigger at K is pulled which opens the bottom of the inner barrel, and the air being suddenly admitted projects the bullet from R with much force.

(32.) *Suppose* the column of mercury to rise from P to Q in the tube.

Then it sinks in the cup to O, a point such that $CO=\frac{1}{17}$ of QP.

Hence the true variation is

$$PQ+\frac{1}{17} \text{ of } PQ, \text{ or, } \frac{18}{17} \text{ of } PQ.$$

Hence in graduating the scale the distances actually measured from the zero point must be less than the space indicated by the numbers placed against the graduations in the ratio of $17:18$.

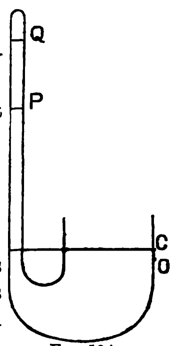

FIG. 194.

(33.) Pressure of air on a square inch

$=$ weight of a column of mercury resting on a square-inch base

$=$ weight of 30·5 cubic inches of mercury

$=$ weight of $(30{\cdot}5 \times 13{\cdot}57)$ cubic inches of water

$= 30{\cdot}5 \times 13{\cdot}57 \times 252{\cdot}6$ grains

$= 104547{\cdot}351$ grains.

Now 7000 grains troy $=$ a pound avoirdupois, and $\therefore$ 104547·351 grains $=$ 14·935 lbs. nearly.

(34.) The pressure of air in the upper part of the tube, when the instrument stands at 27 inches, is equal to the weight of $3a$ cubic inches of mercury, a being the area of the base of the column in the tube.

The air occupies a space of $9a$ cubic inches.

Hence if this air be subjected to a pressure of $30a$ cubic inches of mercury it will be reduced to atmospheric density, and if x be the space it then occupies in cubic inches,

$$x : 9a = 3a : 30a; \quad \text{(HYDROSTATICS, Art. 80.)}$$

$$\therefore x = \frac{27a}{30} = \frac{9a}{10};$$

$\therefore$ the air will occupy $\frac{9}{10}$ of an inch in length.

(35.) Actual weight of stones + weight of air displaced by stones $=$ actual weight of the weights + weight of air displaced by weights.

Now the weights are smaller in volume than the stones.

$\therefore$ weight of air displaced by stones is greater than weight of air displaced by weights.

And a diminution of atmospheric pressure will decrease the weight of air displaced in each case in proportion to the weight of air displaced before in each case.

$\therefore$ it will decrease the weight of air displaced by stones more than it decreases the weight of air displaced by weights.

$\therefore$ the stones will appear to weigh more than their actual weight;

$\therefore$ he will gain by selling when the barometer is low.

(36.) The confined air is pressed by a column of mercury equal to $\frac{1}{2}$ atmospheric pressure, and by the atmospheric pressure at the surface of this mercury.

$\therefore$ we have a body of air at atmospheric density subjected to a pressure $\frac{3}{2}$ times the atmospheric pressure;

$\therefore$ the air will be compressed into $\frac{2}{3}$ of the space it occupied before;

$\therefore$ the mercury descends $\frac{1}{3}$ of 14 inches, or $4\frac{2}{3}$ inches.

EXAMPLES—VI. (p. 75.)

(1.) It will increase the time of filling the receiver, because the only *effective* work will be done by the descending piston after it has passed the hole. Hence, if the aperture be made one-third of the way up the barrel, the distance through which the piston acts effectively being only one-third of the distance through which it worked effectively at first, three times the original time will be required for filling the tank.

(2.) The tension of the piston-rod = pressure of atmosphere above − pressure of atmosphere below.

Now pressure of atmosphere above $= (4 \times 15)$ lbs. $= 60$ lbs., pressure of atmosphere below $= (4 \times 15)$ lbs. − weight of a column of water 16 feet high resting on a base of 4 square inches

$$= 60 \text{ lbs.} - \text{weight of } \left(16 \times \frac{4}{144}\right) \text{ cubic feet of water;}$$

$$\therefore \text{ tension of rod} = \text{weight of } \frac{4}{9} \text{ cubic feet of water.}$$

$$= \left(\frac{4}{9} \times 1000\right) \text{ ounces} = \frac{4 \times 1000}{9 \times 16 \text{ lbs.}}$$

$$= \frac{250}{9} \text{ lbs.} = 27\tfrac{7}{9} \text{ lbs.}$$

(3.) (α) If the hole be made in the longer arm, *below* the level of the shorter arm, no effect will be produced.

(β) If the hole be made in the longer arm, *above* the level of the shorter arm, all the fluid in the longer arm below the hole will descend, and all above in the same branch will ascend, causing the remainder of the fluid to flow through the short branch, till the siphon is emptied.

(γ) If the hole be made in the shorter arm, all the fluid below the hole in this arm will descend; all above in this arm will ascend and flow through the longer arm, emptying the siphon.

(δ) If the hole be made at the top of the siphon, the fluid will descend in each arm, and will empty the siphon.

(4.) A height equal to the height of the water barometer at the time, which is ($13{\cdot}57 \times 29$) inches, or $393{\cdot}53$ inches, that is, 32 feet $9{\cdot}53$ inches, or $32{\cdot}7941\dot{6}$ feet.

(5.) The fluid would descend in each branch, and the siphon would be emptied.

(6.) Equally well at both.

(7.) No; because the water in the hold must be below the level of the surface of the water in the harbour.

(8.) The height of the water barometer at the time, and this is (taking the specific gravity of mercury as $13{\cdot}57$), ($30 \times 13{\cdot}57$) inches, or $407{\cdot}1$ inches, that is, 33 feet $11{\cdot}1$ inches.

(9.) If the air be removed from the siphon, the fluids will first ascend in each arm, and then the flow from the longer arm will commence and go on in the usual manner.

(10.) The water will rise in the inverted tube as high as the top of the inserted tube, and then it will flow out of this tube.

(11.) The water would soon cease to flow, because there would be no atmospheric pressure to cause a continuous flow. But when the air is gradually re-admitted, the water will begin to rise in each branch, and ultimately the flow will go on regularly.

(12.) (*a*) The water will flow into the lower vessel.
(*b*) The water will descend in each arm of the siphon till it stands at a height above each surface equal to the height of the water barometer.
(*c*) The water will flow into the lower vessel.

(13.) The shorter arm must be 2 feet in length, and the longer arm just more than 2 feet.

EXAMPLES—VII. (p. 81.)

(1.) $C=\frac{5}{9}(F-32)$, and $R=\frac{4}{9}(F-32)$;

$$\therefore (1)\ C=\frac{5}{9}(30-32)=\frac{-10}{9}=-1\tfrac{1}{9}.$$

$$R=\frac{4}{9}(30-32)=-\frac{8}{9}.$$

$$(2)\ C=\frac{5}{9}(45-32)=\frac{5}{9}\times 13=7\tfrac{2}{9}.$$

$$R=\frac{4}{9}(45-32)=\frac{4}{9}\times 13=5\tfrac{7}{9}.$$

$$(3)\ C=\frac{5}{9}(56-32)=\frac{5}{9}\times 24=13\tfrac{1}{3}.$$

$$R=\frac{4}{9}(56-32)=\frac{4}{9}\times 24=10\tfrac{2}{3}.$$

$$(4)\ C=\frac{5}{9}(0-32)=\frac{-160}{9}=-17\tfrac{7}{9}.$$

$$R=\frac{4}{9}(0-32)=\frac{-128}{9}=-14\tfrac{2}{9}.$$

$$(5)\ C=\frac{5}{9}(-7-32)=\frac{-195}{9}=-21\tfrac{2}{3}.$$

$$R=\frac{4}{9}(-7-32)=\frac{-156}{9}=-17\tfrac{1}{3}.$$

$$(6)\ C=\frac{5}{9}(-45-32)=\frac{-385}{9}=-42\tfrac{7}{9}.$$

$$R=\frac{4}{9}(-45-32)=\frac{-308}{9}=-34\tfrac{2}{9}.$$

(2.) $C=\frac{5R}{4}$, and $F=\frac{9R}{4}+32$;

$$\therefore (1)\ C=\frac{5\times 5}{4}=\frac{25}{4}=6\tfrac{1}{4}.$$

$$F=\frac{45}{4}+32=11\tfrac{1}{4}+32=43\tfrac{1}{4}.$$

(2) $C=\frac{5\times 20}{4}=5\times 5=25.$

$F=\frac{9\times 20}{4}+32=45+32=77.$

(3) $C=\frac{5\times 0}{4}\times 0.$

$F=0+32=32.$

(4) $C=\frac{-90}{4}=-22\frac{1}{2}.$

$F=\frac{-162}{4}+32=-40\frac{1}{2}+32=-8\frac{1}{2}.$

(5) $C=\frac{5\times(-64)}{4}=5\times(-16)=-80.$

$F=\frac{9\times(-64)}{4}+32=(9\times -16)+32=-112.$

(6) $C=\frac{5\times 120}{4}=150.$

$F=\frac{9\times 120}{4}+32=270+32=302.$

(3) $F=\frac{9}{5}\,C+32$, and $R=\frac{4C}{5}$;

$\therefore$ (1) $F=\frac{9\times 16}{5}+32=28\frac{4}{5}+32=60\frac{4}{5}.$

$R=\frac{4\times 16}{5}=\frac{64}{5}=12\frac{4}{5}.$

(2) $F=\frac{9\times 45}{5}+32=81+32=113.$

$R=\frac{4\times 45}{5}=4\times 9=36.$

(3) $F=\frac{9\times 110}{5}+32=198+32=230.$

$R=\frac{4\times 110}{5}=4\times 22=88.$

(4) $F=\frac{9\times 0}{5}+32=32.$

$R=\frac{4\times 0}{5}=0.$

(5) $F=\frac{9\times(-15)}{5}+32=-27+32=5.$

$R=\frac{4\times(-15)}{5}=-12.$

(6) $F=\frac{9\times(-24)}{5}+32=-\frac{216}{5}+32=-11\frac{1}{5}.$

$R=\frac{4\times(-24)}{5}=-\frac{96}{5}=-19\frac{1}{5}.$

(4.) Yes, if the graduations are to be uniform.

(5.) $\left.\begin{array}{l}9C=5\ (F-32)\\ C+F=60\end{array}\right\};$

$\therefore\ 9C=5\ (60-C-32),$

or, $9C=300-5C-160$;

$\therefore\ 14C=140$, and $\therefore\ C=10$, and $F=50.$

(6.) $\left.\begin{array}{l}9C=5\ (F-32)\\ F=5C\end{array}\right\};$

$\therefore\ 9C=5\ (5C-32)$;

$\therefore\ 16C=160$, and $\therefore\ C=10$, and $F=50.$

(7.) $\left.\begin{array}{l}9C=5\ (F-32)\\ C=F\end{array}\right\};$

$\therefore\ 9C=5\ (C-32)$;

$\therefore\ 4C=-160$, and $\therefore\ C=-40$, and $F=-40.$

(8.) $(80-20)^\circ$, or 60° on the new scale $=(80-32)^\circ$, or $48^\circ F$;

$\therefore$ each degree on the new scale $=\frac{48^\circ}{60}F$, or $\frac{4^\circ}{5}F.$

(9.) $C=\frac{5}{9}\ (F-32)$;

$\therefore\ C=\frac{5}{9}\ (78-32)=\frac{5\times 46}{9}=\frac{230}{9}=25\frac{5}{9}.$

(10.) $\left.\begin{array}{l} 9C=5\,(F-32) \\ C+F=88 \end{array}\right\}$;

$$\therefore\ 9C=5\,(88-C-32)\,;$$

$$\therefore\ 14C=280, \text{ and } \therefore\ C=20, \text{ and } F=68.$$

(11.) $C=\dfrac{5}{9}\,(F-32)$;

$$\therefore\ C=\frac{5}{9}\,(49-32);$$

$$\therefore\ C=\frac{5\times 17}{9}=\frac{85}{9}=9\tfrac{4}{9}.$$

(12.) The graduations would be inconveniently small.

(13.) $\left.\begin{array}{l} 9C=5\,(F-32) \\ F=3C \end{array}\right\}$;

$$\therefore\ 9C=5\,(3C-32)\,;$$

$$\therefore\ 6C=160, \text{ and } \therefore\ C=26\tfrac{2}{3}, \text{ and } F=80.$$

(14.) $\left.\begin{array}{l} 9C=5F-160 \\ 17C=5F \end{array}\right\}$;

$$\therefore\ 9C=17C-160, \text{ and } \therefore\ C=20, \text{ and } F=68.$$

(15.) $\left.\begin{array}{l} 9C=5F-160 \\ -C=F \end{array}\right\}$;

$$\therefore\ 9C=-5C-160, \text{ and } \therefore\ C=-11\tfrac{3}{7}, \text{ and } F=11\tfrac{3}{7}.$$

(16.) Let x be the number of degrees on the latter.

Then $\dfrac{15-9}{15-10}=\dfrac{x-12}{x-14}$;

$$\therefore\ 6\,(x-14)=5\,(x-12)\,;$$

$$\therefore\ 6x-84=5x-60,\ \therefore\ x=24.$$

(17.) Let x be the number of degrees on the latter.

Then $\dfrac{16-8}{16-10}=\dfrac{x-11}{x+14}$;

$$\therefore\ 8\,(x-14)=6\,(x-11)\,;$$

$$\therefore\ 8x-112=6x-66, \text{ and } \therefore\ x=23.$$

(18.) $\left.\begin{array}{l} R=\frac{4}{9}(F-32) \\ R=F-47 \end{array}\right\}$;

$$\therefore 9R=4(R+47-32);$$
$$\therefore 5R=60, \text{ and } \therefore R=12, \text{ and } F=59.$$

Also, if d be the number of degrees in the difference,

$$\left.\begin{array}{l} R=\frac{4}{9}(F-32) \\ R=F-47-d \end{array}\right\};$$

$$\therefore 4F-128=9(F-47-d), \text{ or, } 5F=295+9d, \text{ or, } F=59+\frac{9}{5}d;$$

$$\therefore F \text{ rises } \frac{9d}{5} \text{ and } \therefore R \text{ rises } \frac{4d}{5}.$$

EXAMPLES—VIII. (p. 87.)

1. Let x be the volume of the part immersed, y the volume of the part out of the water.

$$\text{Then } \frac{x}{x+y}=\frac{\cdot925}{1\cdot025};$$
$$\therefore 1025x=925x+925y;$$
$$\therefore 100x=925y;$$
$$\text{and } \therefore y:x=100:925=4:37.$$

2. Let x be the volume of the body, y the volume of the part out of the fluid in the first case, s the specific gravity of the body.

$$\text{Then } \frac{x-y}{x}=\frac{s}{\cdot9},$$
$$\text{and } \frac{y}{x}=\frac{s}{1\cdot1};$$
$$\therefore \frac{x-y}{y}=\frac{1\cdot1}{\cdot9};$$
$$\therefore \frac{x}{y}-1=\frac{11}{9}, \text{ or, } \frac{x}{y}=\frac{20}{9}.$$
$$\text{Hence } \frac{9}{20}=\frac{s}{1\cdot1},$$
$$\text{and } \therefore s=\frac{99}{200}=\cdot495.$$

3. $F = \frac{9C}{5} + 32$;

$$\therefore \text{ when } C = -40^\circ,$$
$$F = -72 + 32 = -40;$$
$$\text{and when } C = 350^\circ,$$
$$F = 630 + 32 = 662.$$

4. After three strokes

$$\text{density of air in receiver} = \left(\frac{3}{1+3}\right)^3 . \text{ original density};$$

$$\therefore \text{ pressure} = \frac{27}{64} \text{ of original pressure};$$

$$\therefore \text{ height of barometer} = \frac{27}{64} \text{ of 28 inches} = 11{\cdot}8125 \text{ inches.}$$

5. Let w_1 and w_2 be the weights of the hydrometers, v the volume of fluid displaced in each case, s_1 and s_2 the specific gravity of the fluids.

Then $\frac{s_1}{s_2} = \frac{vs_1}{vs_2} = \frac{h_1}{h_2} = \frac{1}{p}$.

6. Let v be the volume of the man, s his specific gravity;

$$\therefore \frac{v}{12} \text{ is the volume of his head.}$$

$$\frac{11v}{12} = \text{volume of man immersed,}$$

$$\frac{2v}{12} = \text{volume of bladders};$$

$$\therefore \text{ volume of water displaced} = \frac{13v}{12}.$$

Then $v \cdot s = \frac{13v}{12} \cdot 1$, or, $s = \frac{13}{12} = 1{\cdot}08\dot{3}$.

7. Height $= (28 \times 13{\cdot}6)$ inches $= 380{\cdot}8$ inches $= 31$ feet $8{\cdot}8$ inches.

8. $C = \frac{5}{9}(F - 32) = \frac{5}{9}(27 - 32) = -\frac{25}{9} = -2\frac{7}{9}$.

9. Let x be the depth in feet reached by his fingers,

$x - \frac{15}{2}$ is the depth reached by his feet;

$$\therefore x : x - \frac{15}{2} = 3 : 2,$$

$$\text{or, } 2x = 3x - 22\tfrac{1}{2}, \text{ and } \therefore x = 22\tfrac{1}{2} \text{ feet.}$$

10. Let x be the size of the cork in cubic feet.

Then weight of cork $= 240x$ ounces, and weight of mercury displaced $= 13600x$ ounces;

$$\therefore x\,(13600 - 240) = 167;$$

$$\therefore x = \frac{167}{13360} = \frac{1}{80}.$$

11. Let x be the number of degrees on De Lisle.

$$\text{Then } \frac{150 - x}{150} = \frac{47 - 32}{180},$$

$$\text{or, } \frac{150 - x}{5} = \frac{15}{6};$$

$$\therefore 900 - 6x = 75, \text{ or, } 6x = 825;$$

$$\therefore x = 137\tfrac{1}{2}.$$

12. Let v = volume of cork.

Then since $(\cdot 0013 \times n)$ = specific gravity of n atmospheres,

$$v \times \cdot 24 = nv \times \cdot 0013;$$

$$\therefore n = \frac{\cdot 24}{\cdot 0013} = \frac{2400}{13}.$$

13. The body will rest when reduced to *half* its natural size;

$$\therefore \frac{20 + n}{20 + 4n} = \frac{1}{2};$$

$$\therefore 40 + 2n = 20 + 4n;$$

$$\therefore n = 10 \text{ feet.}$$

14. (a) When it is at depth of 5 feet,

$$\frac{20 + n}{20 + 4n} = \frac{20 + 5}{20 + 20} = \frac{5}{8};$$

$\therefore$ body displaces $\frac{5}{8}$ of 20 lbs. of water, or, $12\frac{1}{2}$ lbs. of water;

$\therefore$ since the body weighs 10 lbs., we must have a downward force of $2\frac{1}{2}$ lbs.

(β) When it is at depth of 30 feet,

$$\frac{20+n}{20+4n}=\frac{20+30}{20+120}=\frac{50}{140}=\frac{5}{14};$$

$\therefore$ it displaces $\frac{5}{14}$ of 20 lbs. of water, or, $7\frac{1}{7}$ lbs. of water,

$\therefore$ we must have an upward force of $2\frac{6}{7}$ lbs.

15. When the body displaces 11 lbs. and 9 lbs. of water respectively.

(1) When $\dfrac{20+n}{20+4n}=\dfrac{11}{20}$,

$$\text{or, } 400+20n=220+44n,$$
$$\text{or, } 24n=180, \text{ and } \therefore\ n=7\tfrac{1}{2} \text{ feet.}$$

(2) When $\dfrac{20+n}{20+4n}=\dfrac{9}{20}$,

$$\text{or, } 400+20n=180+36n,$$
$$\text{or, } 16n=220, \text{ and } \therefore\ n=13\tfrac{3}{4} \text{ feet.}$$

16. Let F be the reading on the Fahrenheit scale.

Then $\frac{4}{9}(F-32)$ is the reading on Reaumur's scale.

$$\therefore F:\frac{4}{9}(F-32)=25:4;$$
$$\therefore 36F=100F-3200.$$
$$\text{Hence } F=50.$$

17. Let F be the reading on the Fahrenheit scale.

Then $\frac{4}{9}(F-32)$ is the reading on Reaumur's scale,

and $\frac{5}{9}(F-32)$ is the reading on the Centigrade scale.

$$\therefore F+\frac{4}{9}(F-32)+\frac{5}{9}(F-32)=212;$$
$$\therefore F+F-32=212;$$
$$\therefore F=122.$$

18. Let w be the apparent weight of each in water.

Then $\frac{2\cdot6}{1}=\frac{22}{22-w}$, (HYDROSTATICS, Art. 65, Case I.)

and $\frac{7\cdot8}{1}=\frac{n}{n-w}$;

$$\left.\begin{aligned}\therefore \frac{22}{2\cdot6}&=22-w,\\ \text{and } \frac{n}{7\cdot8}&=n-w\end{aligned}\right\};$$

$$\therefore \frac{22}{2\cdot6}-\frac{n}{7\cdot8}=22-n;$$

$$\therefore 66-n=7\cdot8\,(22-n);$$

$$\therefore 6\cdot8n=105\cdot6, \text{ and } \therefore n=15\tfrac{9}{17}.$$

19. Let w be the apparent weight of the substances in the fluids, s the specific gravity of the fluid.

Then $\frac{s}{3}=\frac{1-w}{1}$,

and $\frac{s}{2\cdot25}=\frac{3-w}{3}$;

$$\left.\begin{aligned}\therefore \frac{s}{3}&=1-w,\\ \text{and } \frac{3s}{2\cdot25}&=3-w\end{aligned}\right\};$$

$$\therefore \frac{s}{\cdot75}-\frac{s}{3}=2, \text{ or, } 4s-s=6, \text{ and } \therefore s=2.$$

20. Let N be the position of the fulcrum, v the volume of each body.

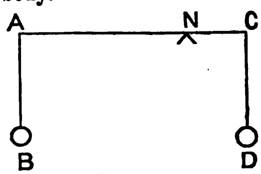

FIG. 195.

Then tension of string $AB=v\,(2\cdot7-1)$,
and tension of string $CD=v\,(6\cdot1-1)$;

$\therefore$ if $CN=x$ inches,

$$\frac{v\,(2\cdot7-1)}{v\,(6\cdot1-1)}=\frac{x}{71-x};$$

$$\therefore \frac{17}{51}=\frac{x}{71-x}, \text{ or, } 71-x=3x;$$

$$\therefore x=\frac{71}{4} \text{ inches}=17\tfrac{3}{4} \text{ inches.}$$

EXAMPLES—IX. (p. 94.)

1. Let x and y be the lengths of the arms at the ends of which the gold and silver hang in each of the three cases: then

(1) $x : y = 10{\cdot}4 : 19{\cdot}4$
$= 104 : 194$
$= 52 : 97.$

(2) $x : y = 10{\cdot}4 - 1 : 19{\cdot}4 - 1$
$= 9{\cdot}4 : 18{\cdot}4$
$= 47 : 92.$

(3) In this case the silver is forced up by the mercury, and acts on the lever vertically upwards, hence the fulcrum must be at one end, and the gold between the silver and the fulcrum, and
$x : y = 13{\cdot}5 - 10{\cdot}4 : 19{\cdot}4 - 13{\cdot}5$
$= 3{\cdot}1 : 5{\cdot}9$
$= 31 : 59.$

2. Let x be the weight of the body in the fluid.

Then $\frac{7-x}{7} = \frac{3}{7}$;

$$\therefore x = 4 \text{ lbs.}$$

Then, by *Statics*, Art. 102,

$$\text{height} : \text{base} = 3 : 4.$$

3. Let v be the volume of hydrogen in the balloon in cubic feet.

Then $v \times {\cdot}07 \times 1{\cdot}3 =$ weight of hydrogen in ounces,

and $v \times 1{\cdot}3 =$ weight of displaced air in ounces;

$$\therefore v \times 1{\cdot}3 = v \times {\cdot}07 \times 1{\cdot}3 + 10 \times 112 \times 16;$$

$$\therefore v \times 1{\cdot}3 \times {\cdot}93 = 10 \times 112 \times 16;$$

$$\therefore v = \frac{10 \times 112 \times 16}{1{\cdot}209} \text{ cubic feet.}$$

4. Let p_1 denote the pressure internally before the ascent, p_2 the pressure internally after the ascent.

$$\text{Then } p_2 : p_1 = 3 : 4.$$

Hence if v_1 and v_2 be the volumes of the gas before and after the ascent,

$$v_2 : v_1 = p_1 : p_2 = 4 : 3.$$

Hence one-third of the gas must have been expelled to maintain equilibrium between the external and internal pressures.

Also, since the pressure of the external air is only three-fourths of the pressure of the external air before the ascent, one-fourth of the whole weight must have been thrown out.

5. The gas has been expelled to preserve equilibrium of internal and external pressures, and the ballast to preserve equilibrium of vertical pressures on the balloon.

6. Volume of cylindrical vessel $= \pi \,.\, 4^2 \,.\, 9$ cubic inches.

Volume of interior of cylinder $= \pi \,.\, 3^2 \,.\, 8$ cubic inches;

$\therefore$ volume of wood $= \pi\,(16 \times 9 - 9 \times 8)$ cubic inches $= 72\,\pi$ cubic inches, also, volume of cylindrical mass of fluid displaced $= \pi \,.\, 4^2 \,.\, 3$ cubic in.;

$\therefore$ if s be the specific gravity of the wood,

$$72\pi \,.\, s = 48\pi;$$

$$\therefore\ s = \frac{48}{72} = \frac{2}{3}.$$

Again, let x be the depth to which it sinks.

Then volume of fluid displaced $= \pi \,.\, 4^2 + \pi\,(4^2 - 3^2)\,(x - 1)$

$$= 7\pi x + 9\pi;$$

$$\therefore\ 7\pi x + 9\pi = 72\pi \,.\, s = 48\pi;$$

$$\therefore\ 7x = 39, \text{ or, } x = 5\tfrac{4}{7} \text{ inches.}$$

7. No change will occur till the stone falls from the ice; but then the ice will displace less water than before, and the surface of the water will sink.

8. Let AB be the surface of the water outside, CD the level to which the water rises inside.

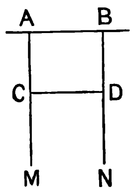

FIG. 196.

Then, by Boyle's law,

pressure of air in AD : atmospheric pressure $= BN : BD$.

Hence if h represent the atmospheric pressure, and x be the length of BD, and y the length of DN,

$$\frac{\text{pressure of air in } AD}{\text{atmospheric pressure}} = \frac{x+y}{x};$$

$$\therefore\ \frac{x+h}{h} = \frac{x+y}{x}, \text{ or, } x^2 + xh = xh + hy, \text{ and } \therefore\ x^2 = hy.$$

9. The pressure is equal to the weight of (80×6) cubic feet of water.

Hence if we take the weight of a cubic foot of sea water as 1000 ounces (which is rather less than the actual weight),

$$\text{pressure} = (80 \times 6 \times 1000) \text{ ounces} = 30000 \text{ lbs.}$$

If the box were not water-tight the water would enter it, and the pressure will be the same inside as outside.

10. Let x be the number of feet to which the bag is sunk.

The pressure on the bag will then be equal to a pressure indicated by a height of $(x+34)$ feet on the water barometer.

$$\therefore \frac{1}{19} = \frac{34}{x+34};$$

$$\therefore x = (18 \times 34) \text{ feet} = 102 \text{ fathoms.}$$

11. Let x be the weight of the vessel in ounces.

Then, since the weight of a cubic foot of water is 1000 ounces,

$$x + \frac{7}{8} \times 1000 = 1000;$$

$$\therefore x = \frac{1000}{8} = 125 \text{ ounces.}$$

12. The surface of the fluid will rise higher than the surface of the water, because the specific gravity of the fluid is less than the specific gravity of the water.

13. Let x be the depth in inches to which the cylinder sinks.

Then $15 \times 1{\cdot}1 + 15 \times {\cdot}25 = x \times 1$;

$$\therefore x = 15 \times 1{\cdot}35 = 20{\cdot}25 \text{ inches.}$$

14. The larger piece of cork will rise to the surface, and the length of the string between it and the pulley will be 2 feet, while the smaller piece of cork, at the end of 1 foot of string from the pulley, will be entirely submerged.

Then if w and $3w$ be the weights of the pieces of cork, the smaller piece displaces water to the weight of $4w$.

Hence we must have an upward force $= 3w$ at end of longer string.

$\therefore$ the larger piece of cork must displace water to the weight of $6w$, and $\therefore$ it must be half immersed.

15. As the air is drawn away from the tube the water will rise in both branches, and if the height of the top of the tube above the surface of the water in the reservoirs is not too great, there will ultimately be a continuous column of water in both parts of the siphon, and then a regular flow will commence.

16. Let v be the volume of each body, s_1 and s_2 the specific gravities of the bodies, ρ_1 and ρ_2 the specific gravities of the liquids.

Then, $s_1 - \rho_1 = s_2 - \rho_2$, or, $s_1 - s_2 = \rho_1 - \rho_2$, ... (1)

And $s_1 - \frac{\rho_2}{2} = s_2 - \rho_1$, or, $s_1 - s_2 = \frac{\rho_2 - 2\rho_1}{2}$ . . . . (2)

$$\text{Hence } \rho_1 - \rho_2 = \frac{\rho_2 - 2\rho_1}{2},$$

$$\text{and thus } \rho_1 = \frac{3\rho_2}{4}, \text{ and } s_1 - s_2 = -\frac{\rho_2}{4}.$$

Let x be the part of the heavier body immersed in the final experiment.

Then $vs_1 = vs_2 - xv\left(\frac{\rho_1 + \rho_2}{2}\right)$;

$$\therefore s_1 - s_2 = -\frac{x}{2} \cdot (\rho_1 + \rho_2);$$

$$\therefore -\frac{\rho_2}{4} = -\frac{x}{2} \cdot \frac{7\rho_2}{4}, \text{ and thus } x = \frac{2}{7}.$$

17. AD is the level of the mercury, R is the height of the water. Then the question to resolve is whether the weight of a column of mercury in height CD in the longer arm is greater (or not) than the weight of a column of water, height CR, on the same base, since the pressure *upwards* at the bottom of this tube = pressure upwards at A + weight of such a column of water;

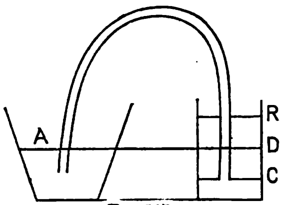

FIG. 197.

$\therefore$ mercury will flow through the tube if $CD \times$ density of mercury be greater than $CR \times$ density of water,

$$\text{or, if } z\rho \text{ be greater than } z'\rho',$$

$$\text{or, } \frac{z}{z'} \text{ be greater than } \frac{\rho'}{\rho}.$$

18. Let x = depth of air at the top of the vessel when the water stands at the top of the pipe.

The amount of water forced in at each stroke is Al.

Then, if π be the atmospheric pressure, and ρ the atmospheric density,

pressure of confined air

$$=\frac{c}{x}\pi=(h-(c-x))\rho+\pi, \text{ and } \pi=\rho h;$$

$$\therefore \frac{ch}{x}=2h-c+x,$$

$$\text{and } \therefore x=\frac{c-2h\pm\sqrt{4h^2+c^2}}{2}.$$

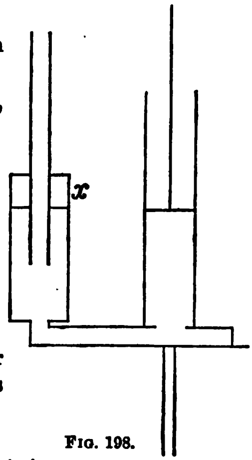

FIG. 198.

Again, considering the quantity of water in the air-vessel and pipe, after n strokes of the piston.

$$Aln=(c-x)A+a(h-c+x);$$

$$\therefore n=\frac{A(2h+c-\sqrt{c^2+4h^2})+a(\sqrt{c^2+4h^2}-c)}{2Al}.$$

19. Let x be the weight of the cylinder in lbs.

Then $x+6:x=8:5$;

$$\therefore 5x+30=8x, \text{ or, } x=10 \text{ lbs.}$$

20. Let s_1 and s_2 be the specific gravities of the metals, v and w the measures of the volumes and weights mixed.

Then $vs_1+vs_2=2v.9$,

$$\text{and } \frac{w}{s_1}+\frac{w}{s_2}=\frac{2w}{8\frac{8}{9}};$$

$$s_1+s_2=18,$$

$$\frac{1}{s_1}+\frac{1}{s_2}=\frac{9}{40},$$

and hence we find $s_1=10$ or 8, and $s_2=8$ or 10.

21. Let x be the height of the cylinder.

Then, since the air which stood at height x inches under the atmospheric pressure is condensed to the height of 1 inch,

$$1:x=15:165;$$

$$\therefore x=\frac{165}{15}=11 \text{ inches.}$$

22. The density of the air in the receiver of the air-pump is $\frac{2}{3}$ of $\frac{2}{3}$, or $\frac{4}{9}$ of what it was at first.

The density of the air in the receiver of the condenser is after the first stroke $\frac{3}{2}$ of what it was at first.

The air in the barrel at the commencement of the second ascent of the piston is $\frac{2}{3}$ of the density of atmospheric pressure.

When the piston has reached a point $\frac{4}{9}$ of the length of the barrel from the valve of the condenser, the air in the barrel will be of the same density as the air in the receiver of the condenser.

The valve will then open, and the air will enter.

The air which occupied a space $\left(1+\frac{2}{9}\right)$ of the receiver will now occupy a space equal to the receiver;

$\therefore$ density will be $\frac{11}{9}$ of the density of air in receiver;

$\therefore$ density will be $\frac{11}{9}$ of $\frac{3}{2}$, or $\frac{33}{18}$ of original density;

$\therefore$ pressure in receiver of condenser : pressure in receiver of air-pump

$$=\frac{33}{18}:\frac{4}{9}$$
$$=33:8.$$

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state of things was for the moment crossed by the death of Alexander (Dec. 1, 1825). The view which his successor Nicholas would take became in the last degree important; Canning, with great wisdom, chose Wellington—opposed indeed to his policy, but personally acceptable to the Russian Czar—as his special ambassador to take the royal congratulations upon the new Emperor's accession, and to continue the negotiations if possible. The appointment met with universal approbation; even Metternich believed that in the hands of Wellington the question must be settled in accordance with his views. It was with much surprise and anger that the Turks and Austrians heard that, on the 4th of April, an arrangement had been arrived at between the Courts of England and Russia. Taking advantage of the very moderate claims of the Greeks, who demanded no more than to be placed on the same footing as the Danubian Principalities, remaining as self-governing but dependent vassals of the Turkish Government, the English minister had succeeded in procuring the signature of a protocol embodying a plan for peaceful intervention.

Protocol between England and Russia. April 1826.

The cause of Greek independence had already excited enthusiasm in England, many volunteers had joined the armies, and money had been subscribed for them. In this enthusiasm Canning in his heart fully joined; from early youth one of his favourite dreams had been the independence of that race to which as an ardent lover of the classics he felt he owed so much. But, true to his principles, and determined to maintain the strict neutrality of England, he had done his best to check any active assistance to the insurgents. According to his view it was necessary that England should intervene with clean hands, and as the friend of both parties. He was also in constant dread of the watchfulness of his Tory enemies, fearing lest any sign of too great favour to Russia should enable them entirely to thwart his plans. Nevertheless the knowledge of the approaching intervention gave a great impetus to the feeling in favour of Greece in England, and men and money were poured in considerable quantities into the peninsula. Lord Cochrane, the most dashing and adventurous of English sailors, had joined the insurgents with an American frigate, General Churchill took command of their armies, yet their destruction seemed immi-

Enthusiasm for Greek independence in England.

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Obs. 1. In the neuters, nom., acc., and voc. are always the same; and in the plural these cases always end in *α*. The contraction of ὀστέα into ὀστᾶ is irregular, cp. 11.

Obs. 2. The following words are feminine:—ὁδός, *way;* νῆσος, *island;* νόσος, *disease;* δρόσος, *dew;* σποδός, *ashes;* ψῆφος, *pebble;* ἄμπελος, *vine;* γνάθος, *jaw;* ἤπειρος, *continent;* and some others.

[Primer of Greek Accidence. *See* p. 24.]

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SHALL AND WILL.

98. In the employment of these words to form a future tense, we must distinguish the *unemphatic* from the *emphatic* use.

In ordinary conversation, when *shall* and *will* are merely used as signs to mark future events, custom (or, as some say, courtesy) has decided that *shall* is to be used for the *first* person, and *will* for the *second* and *third* persons: thus we say

I shall go to London to-morrow.
You will be too late for the train.
The Queen will leave Windsor to-day.

But, even in the discourse of common life, when the *intention* marked by the word *will*, or the *compulsion* implied in the word *shall*, is to be made prominent in even a slight degree, *will* is used with the *first* person, and *shall* with the *second* and *third* persons:

Falstaff. You must excuse me, Master Robert Shallow.

Shallow. I will not excuse you: you shall not be excused: excuses shall not be admitted.

99. Next, in the emphatic language of poetry and the higher prose, *will* denotes *free intention.*

Shall denotes *strong compulsion*, *earnest admonition*, *firm assurance*, what must be, what ought to be, what is sure to come to pass

Hence *will* is often used with the *first* person:

I *will* arise and slay thee with my hands.—*Tennyson.*

And for her sake I do rear up her boy,
And for her sake I *will* not part with him.—*Shakespeare.*

And *shall* is often used with the *second* and *third* persons:

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I.—THE STORY OF ARION.

Arion, after travelling abroad, hires a vessel to take him home.

1.—ARION citharista praeclarus erat. Is diu apud Periandrum Corinthiorum regem versatus erat. Tum in Italiam Siciliamque navigare cupivit. Ingentibus opibus ibi comparatis, Corinthum redire voluit. Itaque Tarento, urbe Italiae, profectus est, ubi navigium hominum Corinthiorum conduxerat.

The sailors form a plan to rob and murder him.

2.—Hi autem eum in mare proiicere constituerunt; pecunia enim potiri cupiebant. Tum vero Arion consilium intellexit. Tristis ad preces confugit. Pecunia omni nautis oblata, vitam deprecatus est. Nautae vero precibus viri non commoti, mortem ei statim minati sunt.

Arion sings a beautiful song, and leaps overboard.

3.—In has angustias redactus Arion, in puppi stetit, omni ornatu suo indutus. Tum unum e carminibus canere incepit. Nautae suavi carmine capti e puppi mediam in navem concesserunt. Ille omni ornatu indutus, capta cithara, carmen peregit. Cantu

NOTES.

PART I.

SIMPLE SENTENCES.

EVERY Simple Sentence is either :—

I. A Statement ; as Psittacus loquitur, *The parrot speaks.*

II. A Command or Request ; as Loquere, psittace, *Speak, parrot.*

III. A Question ; as Loquiturne psittacus ? *Does the parrot speak ?*

1. **apud**—'at the court of.'
   Corinth—a town on the isthmus which separates Northern Greece from the Peloponnesus (island of Pelops).—*Lat. Prim.* § 101.
   ingentibus opibus comparatis.—*Lat. Prim.* § 125.
   Tarentum—now Taranto, the largest Greek city in Italy, on the gulf of the same name.—*Lat. Prim.* § 121, c.
2. **oblata**—from offero.
3. **redactus**—from redigo.
   mediam navem—'the middle of the ship ;' so with other adjectives of position, as, summus mons—'the top of the mountain.'
4. **Taenarum**—now Cape Matapan, the most southern promontory of Greece.
   delatus—from defero.
5. **multum pecuniae**—*lit.* 'much of money.'—*Lat. Prim.* § 131.
6. **Massagetae**—a wandering tribe in Central Asia.
   Scythae—a people of S.-E. Europe.
   simili Scytharum—short for 'like *those* of the S.'
   Utor.—*Lat. Prim.* § 119, a.
   Ex equis—'on horseback.'
   ad omnia—'*for* everything.'
   cocta—from coquo.
7. **quisque . . . sepeliunt**—'They bury . . . each in his own.'
8. **ungulis bovinis**—'with the hoofs of an ox.'—*Lat. Prim.* § 115.
   magnitudine.—*Lat. Prim.* § 116.
9. The phoenix was said to live five hundred years, and then to kill itself by fire, its ashes producing a young one.
   ex intervallo—'after an interval.'
   aliorum . . . aliorum—of some . . . of others.—See 91, note.
   circumlitum—from circumlino.
   magni—'at a high price.'—*Lat. Prim.* § 128. a.

Misled by the absence of allusion to any intention of going against the king, the soldiers applaud. Clearchus' understanding with Cyrus.

7. Ταῦτα εἶπεν· οἱ δὲ στρατιῶται, οἵ τε αὐτοῦ ἐκείνου καὶ οἱ ἄλλοι, ταῦτα ἀκούσαντες, ὅτι οὐ φαίη[63] παρὰ βασιλέα πορεύεσθαι, ἐπῄνεσαν· παρὰ δὲ Ξενίου καὶ Πασίωνος πλείους ἢ δισχίλιοι λαβόντες τὰ ὅπλα καὶ τὰ σκευοφόρα ἐστρατοπεδεύσαντο παρὰ Κλεάρχῳ. 8. Κῦρος δὲ τούτοις[19a] ἀπορῶν τε καὶ λυπούμενος μετεπέμπετο τὸν Κλέαρχον· ὁ δὲ ἰέναι μὲν οὐκ ἤθελε, λάθρᾳ δὲ τῶν στρατιωτῶν[26] πέμπων αὐτῷ ἄγγελον ἔλεγε θαρρεῖν ὡς καταστησομένων τούτων[27, 58a] εἰς τὸ δέον· μεταπέμπεσθαι δ' ἐκέλευεν αὐτόν· αὐτὸς δ' οὐκ ἔφη ἰέναι. 9. Μετὰ δὲ ταῦτα συναγαγὼν τούς θ' ἑαυτοῦ στρατιώτας καὶ τοὺς προσελθόντας αὐτῷ καὶ τῶν ἄλλων[21a] τὸν βουλόμενον ἔλεξε τοιάδε·

Clearchus' second speech. *'Plainly the connexion between us and Cyrus is broken off; I am ashamed to face him, for I fear lest he should punish my breach of faith. Indeed we had all better look out for some way of escape, for Cyrus is a stern foe, and has a large force encamped at our side.'*

"Ἄνδρες στρατιῶται, τὰ μὲν δὴ Κύρου[8a] δῆλον ὅτι οὕτως ἔχει πρὸς ἡμᾶς, ὥσπερ τὰ ἡμέτερα πρὸς ἐκεῖνον· οὔτε γὰρ ἡμεῖς ἐκείνου ἔτι στρατιῶται, ἐπεί γε[51] οὐ συνεπόμεθα αὐτῷ, οὔτε ἐκεῖνος ἔτι ἡμῖν μισθοδότης· ὅτι μέντοι ἀδικεῖσθαι[43a] νομίζει ὑφ' ἡμῶν, οἶδα· 10. ὥστε καὶ μεταπεμπομένου αὐτοῦ[27] οὐκ ἐθέλω[49a] ἐλθεῖν, τὸ μὲν μέγιστον,[14b] αἰσχυνόμενος, ὅτι σύνοιδα ἐμαυτῷ πάντα ἐψευσμένος[9a, 43c] αὐτὸν, ἔπειτα δὲ καὶ δεδιὼς, μὴ λαβών με δίκην ἐπιθῇ[43d] ὧν[4a, 18] νομίζει ὑπ' ἐμοῦ ἠδικῆσθαι. 11. Ἐμοὶ οὖν δοκεῖ οὐχ ὥρα[43a, Obs.] εἶναι ἡμῖν καθεύδειν, οὐδ' ἀμελεῖν ἡμῶν αὐτῶν,[23] ἀλλὰ βουλεύεσθαι, ὅ τι χρὴ[45] ποιεῖν ἐκ τούτων. Καὶ ἕως γε μένομεν[52c] αὐτοῦ, σκεπτέον[31a] μοι δοκεῖ εἶναι, ὅπως ἀσφαλέστατα μενοῦμεν·[50c] εἴ τε ἤδη δοκεῖ ἀπιέναι, ὅπως ἀσφαλέστατα ἄπιμεν, καὶ ὅπως τὰ ἐπιτήδεια ἕξομεν· ἄνευ γὰρ τούτων οὔτε στρατηγοῦ[21] οὔτε ἰδιώτου ὄφελος οὐδέν. 12. Ὁ δ' ἀνὴρ πολλοῦ[26a] μὲν ἄξιος φίλος, ᾧ ἂν φίλος ᾖ,[43] χαλεπώτατος δ' ἐχθρὸς, ᾧ ἂν πολέμιος ᾖ

ἀλεξήσασθαι is not the usual Attic form of the aorist of *ἀλέξω*, but has here the strongest MS. authority.

7. παρὰ βασιλέα] To the king's court; *ἐπὶ*, which would imply hostility, seems purposely avoided. The effect of the speech is plain. Clearchus is not personally popular, but his declaration, that he is not going this long march inland in a strange country, at once brings over some even of Xenias' men, who probably knew what the march was. Ep. i. 1. 2.

8. τούτων] Neuter; that things would right themselves.

9. τὰ μὲν δὴ Κύρου] Cyrus' relations to us must vary with our relation to him. Note the cleverness with which the different points in this speech are put:—1. Of course our pay ceases, and we are thrown on our own resources: 2. we are the aggressors; I cannot face Cyrus, because I know I am treating him shabbily: 3. we shall require all our vigilance to guard our own safety: 4. we cannot neglect the strong force which Cyrus has, and which is sufficient to crush us, for he will be no relenting foe, if foe we make him, and he is close at our doors.

11. ἤδη] At once.

τούτων] *i.e. τῶν ἐπιτηδείων.*

12. ἐχθρὸς] Note the difference between *ἐχθρὸς* and *πολέμιος*. A man may be at war with you without any personal feeling of enmity, but he, if he be your foe, will be a bitter and unrelenting one. Krüger quotes appropriately CURTIUS vii. 10. 8: 'Illi nunquam se *inimicos* ei, sed bello lacessitos *hostes* fuisse, respondent.'

13. ἃ ἐγίγνωσκον] Like the *γνώμην ἀποφαίνεσθαι* of the Athenian assembly.

γνώμης] Consent.

14. εἷς δὲ δὴ εἶπε] 'One went so far as to say.'

ἡ δὲ ἀγορὰ *κ.τ.λ.*] It is this that gives point to the recommendation to buy provisions; it reminds them that they could not even get them without Cyrus' permission.

διὰ φιλίας τῆς χώρας] Note that *φιλίας* is predicate. The presence of a guide from Cyrus might secure their being unmolested.

ὧν πολλοὺς *κ.τ.λ.*] Another insidious hint of danger.

It was the Greeks mainly who had plundered the country in reprisals for the loss of their comrades.

15. ὡς δε] *i.e. ἕκαστος δὲ λεγέτω ὡς.* The construction is changed from *ὡς πεισόμενον*, and a general positive word is understood from the negative *μηδείς.*

16. ὥσπερ *κ.τ.λ.*] As if Cyrus would not want his ships to convey back

[XENOPHON'S ANABASIS OF CYRUS—TAYLOR. *See* p. 30.]

ATTRIBUTIVE EXPRESSIONS.

NOTE 3.—The Objective Genitive in Latin, denoting the object of an action implied in the noun that it qualifies, is often used in phrases where in English we use the Prepositions *for*, *about*, *from*.

ENGLISH.	LATIN.
Resentment *for* a wrong.	Dolor injuriae.
Escape *from* danger.	Fuga periculi.
A craving *for* gain.	Fames lucri.
Sleep is a refuge *from* all toils.	Somnus est perfugium omnium laborum.
Anxiety *about* the body.	Cura corporis.

NOTE 4.—The Attributive Adjective is used in Latin in many cases where we use Prepositions, such as *of*, *in*, *against*; thus—

Mons summus, *the top of the mountain.*
Sullanus exercitus, *the army of Sulla.*
Media aestas, *the middle of the summer.*
Bellum Africanum, *the war in Africa.*
Bellum Mithridaticum, *the war against Mithridates.*
Reliqua Graecia, *the rest of Greece.*
Italia tota, *the whole of Italy.*

NOTE 5.—Observe carefully the following distinctions:—

LATIN.	ENGLISH.
Urbs Roma.	The city *of* Rome.
Sardinia insula.	The island *of* Sardinia.
Civis Romanus.	A citizen *of* Rome.
Civis Atheniensis.	A citizen *of* Athens.
Graecus homo.	A Greek.
Homo Romanus.	A Roman.
Vir patricius.	A patrician.

NOTE 6.—The Objective Genitive follows many adjectives in Latin to express the object of *desire*, *knowledge*, etc., implied in the adjective; thus—

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